

On the Feasibility of Interference Alignment for MIMO Interference Broadcast Channels

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Abstract

In this paper, we analyze the feasibility of linear interference alignment (IA) for multi-input-multi-output (MIMO) interference broadcast channel (MIMO-IBC) without symbol extension. We pose and prove the necessary conditions of IA feasibility for general MIMO-IBC. Except for the proper condition, we find another condition necessary to ensure a kind of *irreducible interference* to be eliminated. We then prove necessary and sufficient conditions for a special class of MIMO-IBC, where the numbers of antennas are divisible by the number of data streams per user. Starting from analyzing the similarities and differences between MIMO-IBC and MIMO interference channel (MIMO-IC), we prove the sufficiency for MIMO-IBC by solving the confliction between eliminating the *reducible interference* and the repeated nature of Jacobian matrix. We show that for the MIMO-IBC where each user has one desired data stream, a proper system is feasible. Finally, we provide proper but infeasible region of antenna configurations for symmetric MIMO-IBC, through analyzing different necessary and sufficient conditions.

Index Terms

Interference alignment feasibility, interference broadcast channel, MIMO, Degrees of freedom (DoF)

I. INTRODUCTION

Inter-cell interference (ICI) is a bottleneck for future cellular networks to achieve high spectral efficiency, especially for multi-input-multi-output (MIMO) systems. When multiple base stations (BSs) can share both the data and the channel state information (CSI), network MIMO can improve the throughput remarkably [1]. When only CSI is shared, the BSs can coordinate to avoid the ICI.

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In information theoretic terminology, the scenario without the data sharing is a MIMO interference broadcast channel (MIMO-IBC) when each BS transmits to multiple users in its serving cell with same time-frequency resource, and is a MIMO interference channel (MIMO-IC) when each BS transmits to one user in its own cell.

To exploit the potential of the interference networks, significant research efforts have been devoted to find the capacity region, which is a very challenging problem. To capture the essential nature of the interference channels, various approaches have been proposed to characterize the capacity. Degrees of freedom (DoF) is the first order approximation of sum rate capacity at high signal-to-noise ratio regime and also called as multiplexing gain, which has received considerable attentions. In a G -cell MIMO-IC where each BS and each user have M antennas, an overall DoF of $GM/2$ can be achieved [2], when using the break-through concept of interference alignment (IA) [3]. In a two-cell MIMO-IBC where each BS and each user have $M = K + 1$ antennas, the system can achieve a DoF of $2M$, as the number of active users in each cell, K , approaches to infinity [4]. This result is surprising, because the DoF is the same as the maximal DOF achievable by network MIMO but without data sharing among the BSs. Encouraged by such a promising performance, many recent works strived for analyzing the DoF for MIMO-IC [5] and MIMO-IBC [6], [7], [8].

With linear strategies, the DoF reflects the dimension of transmit subspace. Therefore, the analysis of feasible conditions for interference-free transmission is crucial to derive the maximum achievable DoF. Yet the feasibility analysis of linear IA for general MIMO-IC and MIMO-IBC is still an open problem since has been recognized in [9].

For MIMO-IC without symbol extension over time or frequency domain, the authors in [10] first related the feasibility of linear IA to the problem of determining the solvability of a system represented by multivariate polynomial equations. By counting the number of equations and independent variables involved in arbitrary subsets of the equations, a *proper condition* was recognized as necessary for the IA feasibility. When the coefficient matrices of the equations are *generic*, the proper system was proved to be feasible in probabilistic (i.e., the system is feasible for *almost all* channel realizations) [10].

To ensure the coefficient matrices to be *generic*, (i) the channel matrix between each BS and each user should be generic, this is true when the elements of the channel matrix are independent and identically distributed (i.i.d.), and (ii) each channel matrix occurs once in the polynomial equations. As pointed out in [10], the coefficient matrices are generic only for the single-beam

MIMO-IC where each BS transmits only one data stream. For the multi-beam case where each BS transmits multiple data streams, the coefficient matrices are not generic any more since the channel matrices repeated several times in the polynomial equations, which satisfy (i) but not (ii). Fortunately, the authors in [11] proved that the proper condition is always necessary for interference-free transmission if only (i) is satisfied. For a special class of multi-beam MIMO-IC where the numbers of transmit and receive antennas are divisible by the number of data streams of each user, the sufficient conditions for the IA feasibility were proved in [11]. For another class of multi-beam MIMO-IC where the numbers of transmit and receive antennas are identical, the sufficient conditions were proved in [12].

For symmetric MIMO-IBC¹, the authors in [13] proposed a necessary condition of IA feasibility by comparing the total number of variables and the total number of equations. By using the same approach, a necessary condition was proposed for a general MIMO-IBC in [14]. For a partially-connected symmetric MIMO-IBC, the authors in [15] provided more necessary conditions, and conjectured that the system is feasible when it is proper if M and N are divisibly by d . However, the rigorous proof in the feasibility of interference-free transmission (i.e., necessary and sufficient conditions) for MIMO-IBC remains unsolved until now.

MIMO-IBC differs from MIMO-IC in system structure. Since in MIMO-IBC each BS supports multiple users, some channel matrices occur more than once in the interference-free transmission equations (which are polynomial equations) even for the single-beam case. This leads to non-*generic* coefficient matrices of the equations because (i) is satisfied but (ii) is not. It indicates that the proper condition may still be one of the necessary conditions for MIMO-IBC.

On the other hand, MIMO-IBC seems similar to multi-beam MIMO-IC. In both systems, each BS transmits multiple data streams, either to multiple users or to a single user in one cell. As a result, each BS generates multiple ICIs to the users in other cells. Some of the ICIs may be avoided by the BS, and others can first be aligned by the BS then be canceled by the user. In other words, each BS may need to use a hybrid mechanism of ICI avoidance and alignment. This is very different from single beam MIMO-IC, where each BS only needs to deal with one ICI, multiple BSs align

¹In a symmetric case, all BSs have the same number of antennas M , all users have the same number of antennas N , the number of users in each cell is identical to K and the number of data streams of each user is identical to d .

the ICI and then each user removes the ICI with reduced dimension.² The hybrid ICI management mechanism in MIMO-IBC introduces new necessary condition, and leads to the challenge in the proof of sufficient condition to ensure interference-free transmission.

In this paper, we investigate the feasibility of linear IA for the MIMO-IBC with constant channels, i.e., we do not consider symbol extension. First, the necessary conditions of interference-free transmission for a general MIMO-IBC will be provided and proved. We find that except for the proper condition, there exist another kind of necessary condition, which ensures that a sort of *irreducible ICI* can be eliminated. Then, the sufficient conditions of interference-free transmission for a special class of MIMO-IBC will be proved, where the numbers of transmit and receive antennas are all divisible by the number of data streams of each user d . This is accomplished by reveal the similarities and differences between the MIMO-IBC and multi-beam MIMO-IC. The proof in sufficiency will be further extended into a more general MIMO-IBC, where the numbers of antennas may not be divisible by d . From the insight provided by the two theorems, we obtain proper but infeasible region of antenna configuration for symmetric MIMO-IBC.

The rest of the paper is organized as follows. We describe the system model in Section II. The necessary conditions for general MIMO-IBC and the necessary and sufficient conditions for a special class of MIMO-IBC will be provided and proved in Section III and Section IV, respectively. We discuss the connection between proper condition and feasible condition in Section V. Conclusions are given in the last section.

Notations: Conjugation, transpose, Hermitian transpose, and expectation are represented by $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, and $\mathbb{E}\{\cdot\}$, respectively. $\text{Tr}\{\cdot\}$ is the trace of a matrix, and $\text{diag}\{\cdot\}$ is a block diagonal matrix. \otimes is the Kronecker product operator, $\text{vec}\{\cdot\}$ is the operator that converts a matrix or set into a column vector, \mathbf{I}_d is an identity matrix of size d . $|\cdot|$ is the cardinality of a set, \emptyset denotes an empty set, and $\mathcal{A} \setminus \mathcal{B} = \{x \in \mathcal{A} | x \notin \mathcal{B}\}$ denotes the relative complement of \mathcal{A} in \mathcal{B} . \exists means “there exists” and \forall means “for all”.

²To realize IA, the BSs and the users should “work together” to remove all the ICI by sharing their spatial resources (the variables in the transmit and receive vectors). This can be observed from the IA feasibility condition of the single beam proper systems (which depends on the summation of the numbers of transmit and receive antennas)[10], and from the proof of feasibility [11]. By contrast, for the coordinated-multipoint coordinated beamforming (CoMP-CB) or multi-user detection (MUD) under zero-forcing (ZF) principle, each BS or each user has enough antennas to eliminate all the ICI. In a more rigorous sense, they are not IA since the ICIs are not compressed into a reduced subspace.

II. SYSTEM MODEL

Consider a downlink G -cell MIMO network. In cell i , BS $_i$ supports K_i users, $i = 1, \dots, G$. The k th user in cell i (denoted by MS $_{i_k}$) is equipped with N_{i_k} antennas to receive d_{i_k} desired data streams from BS $_i$, $k = 1, \dots, K_i$. BS $_i$ is equipped with M_i antennas to transmit overall $d_i = \sum_{k=1}^{K_i} d_{i_k}$ data streams. The total DoF to be supported by the network is $d^{\text{tot}} = \sum_{i=1}^G d_i = \sum_{i=1}^G \sum_{k=1}^{K_i} d_{i_k}$. Assume that there are no data sharing among the BSs and every BS has perfect CSIs of all links. This is a scenario of general MIMO-IBC, and the configuration is denoted by $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, d_{i_k}))$.

The desired signal of MS $_{i_k}$ can be estimated as

$$\hat{\mathbf{x}}_{i_k} = \mathbf{U}_{i_k}^H \mathbf{H}_{i_k,i} \mathbf{V}_{i_k} \mathbf{x}_{i_k} + \sum_{l=1, l \neq k}^{K_i} \mathbf{U}_{i_k}^H \mathbf{H}_{i_k,i} \mathbf{V}_{i_l} \mathbf{x}_{i_l} + \sum_{j=1, j \neq i}^G \mathbf{U}_{i_k}^H \mathbf{H}_{i_k,j} \mathbf{V}_j \mathbf{x}_j + \mathbf{U}_{i_k}^H \mathbf{n}_{i_k} \quad (1)$$

where $\mathbf{x}_{i_k} \in \mathbb{C}^{d_{i_k} \times 1}$ is the symbol vector for MS $_{i_k}$ satisfying $E\{\mathbf{x}_{i_k}^H \mathbf{x}_{i_k}\} = P d_{i_k}$, P is the transmit power per symbol, and $\mathbf{x}_j = [\mathbf{x}_{j_1}^T, \dots, \mathbf{x}_{j_{K_j}}^T]^T$ is the symbol vector for the K_j users in cell j , $\mathbf{V}_{i_k} \in \mathbb{C}^{M_i \times d_{i_k}}$ is the transmit matrix for MS $_{i_k}$ satisfying $\text{Tr}\{\mathbf{V}_{i_k}^H \mathbf{V}_{i_k}\} = d_{i_k}$, and $\mathbf{V}_j = [\mathbf{V}_{j_1}, \dots, \mathbf{V}_{j_{K_j}}]$ is the transmit matrix of BS $_j$ for the K_j users in cell j , $\mathbf{U}_{i_k} \in \mathbb{C}^{N_{i_k} \times d_{i_k}}$ is the receive matrix for MS $_{i_k}$, $\mathbf{H}_{i_k,j} \in \mathbb{C}^{N_{i_k} \times M_j}$ is the channel matrix of the link from BS $_j$ to MS $_{i_k}$ whose elements are i.i.d. random variables with a continuous distribution, $\mathbf{n}_{i_k} \in \mathbb{C}^{N_{i_k} \times 1}$ is an additive white Gaussian noise.

The received signal of each user contains the multiuser interference (MUI) from its desired BS and ICI from its interfering BSs, which are the second and third terms in (1). From (1), the *interference-free transmission equations* are,

$$\text{rank}(\mathbf{U}_{i_k}^H \mathbf{H}_{i_k,i} \mathbf{V}_{i_k}) = d_{i_k}, \forall i, k \quad (2a)$$

$$\mathbf{U}_{i_k}^H \mathbf{H}_{i_k,i} \mathbf{V}_{i_l} = \mathbf{0}, \forall k \neq l \quad (2b)$$

$$\mathbf{U}_{i_k}^H \mathbf{H}_{i_k,j} \mathbf{V}_j = \mathbf{0}, \forall i \neq j \quad (2c)$$

The polynomial equation (2a) is a rank constraint to convey the desired signals for each user. It can be interpreted as a constraint in single user MIMO system, inter-data stream interference (IDI)-free transmission constraint. (2b) and (2c) are the ZF constraints to eliminate the MUI and ICI, respectively.

Note that multiple data streams transmitted from one BS to a user undergo the same channel.³ This leads to two features of MIMO-IBC, according to the cases where the BS and the user are located in the same cell or different cell, which can be observed from the receive signal model.

- *Feature 1*: the desired signal and the MUI experienced at each user undergoing the same channel.
- *Feature 2*: the multiple ICIs generated from one BS for transmitting the desired signals to its own users to a user in other cell undergo the same channel even when each user only receives one desired data stream.

As a result, the coefficient matrices in (2a)-(2c) are not generic.

III. NECESSARY CONDITIONS

In this section, we present and prove the necessary conditions for supporting interference-free transmission in general MIMO-IBC. We start from analyzing (2a)-(2c). Since the conditions are similar to MIMO-IC and the proof builds upon the line of the work in [11], we emphasize the difference of MIMO-IBC from MIMO-IC, which comes from the first feature of MIMO-IBC.

Theorem 1 (Necessary Conditions): In a general MIMO-IBC with configuration $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, d_{i_k}))$ where the channel matrices $\{\mathbf{H}_{i_k,j}\}$ are generic (i.e., drawn from a continuous probability distribution), to ensure interference-free transmission, the following conditions must be satisfied,

$$\min\{M_i - d_i, N_{i_k} - d_{i_k}\} \geq 0, \forall i, k \quad (3a)$$

$$\sum_{j:(i,j) \in \mathcal{I}} (M_j - d_j) d_j + \sum_{i:(i,j) \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} (N_{i_k} - d_{i_k}) d_{i_k} \geq \sum_{(i,j) \in \mathcal{I}} d_j \sum_{k \in \mathcal{K}_i} d_{i_k}, \forall \mathcal{I} \subseteq \mathcal{J} \quad (3b)$$

$$\max \left\{ \sum_{j \in \mathcal{I}_A} M_j, \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} N_{i_k} \right\} \geq \sum_{j \in \mathcal{I}_A} d_j + \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} d_{i_k}, \forall \mathcal{I}_A \cap \mathcal{I}_B = \emptyset \quad (3c)$$

where $\mathcal{K}_i \subseteq \{1, \dots, K_i\}$ is an arbitrary subset of the users in cell i , $\mathcal{J} = \{(i, j) | 1 \leq i \neq j \leq G\}$ denotes the set of all cell-pairs that mutually interfering each other, \mathcal{I} is an arbitrary subset of \mathcal{J} , and $\mathcal{I}_A, \mathcal{I}_B \subseteq \{1, \dots, G\}$ are arbitrary two subsets of the set containing all the G cells.

³Multi-beam MIMO-IC also has such a feature, but for MIMO-IBC this is true for both single beam and multi-beam cases.

A. Proof of (3a)

Proof: Comparing (2a) and (2b), we can see that for MS_{i_k} , the channel matrices of the two equations are all equal to $\mathbf{H}_{i_k,i}$. As a result, the rank constraint is coupled with the MUI-free constraint, such that the proof for MIMO-IC in [9], [10] cannot be directly applied.

To circumvent this problem, note that from the view of MS_{i_k} , the desired data streams of other users in cell i are its MUI, as shown in (1). On the other hand, from the view of BS_i , all the data streams for the users in cell i are all its desired signals. Combining (2a) and (2b), we can obtain a rank constraint for BS_i as

$$\begin{aligned} & \text{rank} \left([\mathbf{H}_{i_1,i}^H \mathbf{U}_{i_1}, \dots, \mathbf{H}_{i_{K_i},i}^H \mathbf{U}_{i_{K_i}}]^H \mathbf{V}_i \right) \\ &= \text{rank}(\text{diag}\{\mathbf{U}_{i_1}^H \mathbf{H}_{i_1,i} \mathbf{V}_{i_1}, \dots, \mathbf{U}_{i_{K_i}}^H \mathbf{H}_{i_{K_i},i} \mathbf{V}_{i_{K_i}}\}) = \sum_{k=1}^{K_i} d_{i_k} = d_i \end{aligned}$$

Then, the interference-free transmission equations in (2a)–(2c) can be equivalently rewritten as

$$\text{rank} \left(\begin{bmatrix} \mathbf{U}_{i_1}^H & \mathbf{0} \\ & \ddots \\ \mathbf{0} & \mathbf{U}_{i_{K_i}}^H \end{bmatrix} \begin{bmatrix} \mathbf{H}_{i_1,i} \\ \vdots \\ \mathbf{H}_{i_{K_i},i} \end{bmatrix} \mathbf{V}_i \right) = d_i, \forall i \quad (4a)$$

$$\mathbf{U}_{i_k}^H \mathbf{H}_{i_k,j} \mathbf{V}_j = \mathbf{0}, \forall i \neq j \quad (4b)$$

Now the channel matrices in (4a) are independent of those in (4b). Since the channel matrix $\mathbf{H}_{i_k,i}$ is generic, (4a) is automatically satisfied with probability one when $\text{rank}(\mathbf{V}_i) = d_i$ and $\text{rank}(\mathbf{U}_{i_k}) = d_{i_k}$ [9], [10]. Therefore, (3a) is necessary to satisfy the equivalent rank constraint (4a). ■

The intuitive meaning of this condition is that BS_i should have enough antennas to transmit the overall d_i desired signals to multiple users in cell i , i.e., to ensure MUI-free transmission, and MS_{i_k} should have enough antennas to receive its d_{i_k} desired signals, i.e., to ensure IDI-free transmission.

B. Proof of (3b)

Proof: To satisfy (4b) under the constraint of (4a), we need to first reserve some variables in the transmit and receive matrices to ensure the equivalent rank constraints, and then use the residual variables to remove the ICI. To this end, we partition the transmit and receive matrices as

follows

$$\mathbf{V}_j = \mathbf{P}_j^V \begin{bmatrix} \mathbf{I}_{d_j} \\ \bar{\mathbf{V}}_j \end{bmatrix} \mathbf{Q}_j^V, \quad \mathbf{U}_{i_k} = \mathbf{P}_{i_k}^U \begin{bmatrix} \mathbf{I}_{d_{i_k}} \\ \bar{\mathbf{U}}_{i_k} \end{bmatrix} \mathbf{Q}_{i_k}^U$$

where $\mathbf{P}_j^V \in \mathbb{C}^{M_j \times M_j}$ and $\mathbf{P}_{i_k}^U \in \mathbb{C}^{N_{i_k} \times N_{i_k}}$ are square permutation matrices, $\mathbf{Q}_j^V \in \mathbb{C}^{d_j \times d_j}$ and $\mathbf{Q}_{i_k}^U \in \mathbb{C}^{d_{i_k} \times d_{i_k}}$ are invertible matrices, and $\bar{\mathbf{V}}_j \in \mathbb{C}^{(M_j - d_j) \times d_j}$ and $\bar{\mathbf{U}}_{i_k} \in \mathbb{C}^{(N_{i_k} - d_{i_k}) \times d_{i_k}}$ are *effective transmit and receive matrices*, whose elements are the residual variables after extracting d_j^2 and $d_{i_k}^2$ variables of \mathbf{V}_j and \mathbf{U}_{i_k} , respectively.

Then, (4b) can be rewritten as

$$(\mathbf{Q}_{i_k}^U)^H \begin{bmatrix} \mathbf{I}_{d_{i_k}} & \bar{\mathbf{U}}_{i_k}^H \end{bmatrix} \underbrace{(\mathbf{P}_{i_k}^U)^H \mathbf{H}_{i_k,j} \mathbf{P}_j^V}_{\bar{\mathbf{H}}_{i_k,j}} \begin{bmatrix} \mathbf{I}_{d_j} \\ \bar{\mathbf{V}}_j \end{bmatrix} \mathbf{Q}_j^V = \mathbf{0} \quad (5)$$

where $\bar{\mathbf{H}}_{i_k,j} = (\mathbf{P}_{i_k}^U)^H \mathbf{H}_{i_k,j} \mathbf{P}_j^V$ is *effective channel matrix*.

Further partition the effective channel matrix as follows

$$\bar{\mathbf{H}}_{i_k,j} = \begin{bmatrix} \bar{\mathbf{H}}_{i_k,j}^{(1)} & \bar{\mathbf{H}}_{i_k,j}^{(2)} \\ \bar{\mathbf{H}}_{i_k,j}^{(3)} & \bar{\mathbf{H}}_{i_k,j}^{(4)} \end{bmatrix}$$

where $\bar{\mathbf{H}}_{i_k,j}^{(1)} \in \mathbb{C}^{d_{i_k} \times d_j}$, $\bar{\mathbf{H}}_{i_k,j}^{(2)} \in \mathbb{C}^{d_{i_k} \times (M_j - d_j)}$, $\bar{\mathbf{H}}_{i_k,j}^{(3)} \in \mathbb{C}^{(N_{i_k} - d_{i_k}) \times d_j}$ and $\bar{\mathbf{H}}_{i_k,j}^{(4)} \in \mathbb{C}^{(N_{i_k} - d_{i_k}) \times (M_j - d_j)}$, respectively.

Then, (5) is equivalent to the following equation,

$$\begin{bmatrix} \mathbf{I}_{d_{i_k}} & \bar{\mathbf{U}}_{i_k}^H \end{bmatrix} \begin{bmatrix} \bar{\mathbf{H}}_{i_k,j}^{(1)} & \bar{\mathbf{H}}_{i_k,j}^{(2)} \\ \bar{\mathbf{H}}_{i_k,j}^{(3)} & \bar{\mathbf{H}}_{i_k,j}^{(4)} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{d_j} \\ \bar{\mathbf{V}}_j \end{bmatrix} = \mathbf{0} \quad (6)$$

which turns into a single interference-free transmission equation that combines (4a) and (4b).

From (6), the relationship between the effective transmit and receive matrices and the effective channel matrices can be expressed in an implicit function form, i.e.,

$$\begin{aligned} \mathbf{F}_{i_k,j}(\bar{\mathbf{H}}; \bar{\mathbf{V}}, \bar{\mathbf{U}}) &= \bar{\mathbf{H}}_{i_k,j}^{(1)} + \bar{\mathbf{H}}_{i_k,j}^{(2)} \bar{\mathbf{V}}_j + \bar{\mathbf{U}}_{i_k}^H \bar{\mathbf{H}}_{i_k,j}^{(3)} + \bar{\mathbf{U}}_{i_k}^H \bar{\mathbf{H}}_{i_k,j}^{(4)} \bar{\mathbf{V}}_j \\ &= \mathbf{0}, \quad \forall i \neq j \end{aligned} \quad (7)$$

where $\mathbf{F}_{i_k,j}(\cdot)$ represents the ICI from $\bar{\mathbf{V}}_j$ to $\bar{\mathbf{U}}_{i_k}$, i.e., the interference generated by the effective transmit matrix of BS_j to the *k*th user in cell *i*.

$\mathbf{F}_{i_k,j}(\cdot) \in \mathbb{C}^{d_{i_k} \times d_j}$ includes $d_j d_{i_k}$ ICIs, and $\bar{\mathbf{V}}_j \in \mathbb{C}^{(M_j - d_j) \times d_j}$ and $\bar{\mathbf{U}}_{i_k} \in \mathbb{C}^{(N_{i_k} - d_{i_k}) \times d_{i_k}}$ provide $(M_j - d_j)d_j$ and $(N_{i_k} - d_{i_k})d_{i_k}$ variables, respectively. Hence, (3b) ensures that for all subsets of

the equations in (7), the number of the variables involved is at least as large as the number of the corresponding equations. Analogous to MIMO-IC, to eliminate all the ICI, all the cell-pairs that interfering each other should be considered, i.e., those in set \mathcal{J} and any subset of it \mathcal{I} , should be considered. Different from MIMO-IC, to ensure ICI-free transmission for every user, we should not generate ICI to arbitrary subsets of the users in each cell, i.e., \mathcal{K}_i , rather than a single user or all users in each cell.⁴

From the definition in [10], we know that this is actually the condition to ensure the system to be proper. For MIMO-IC with generic channel matrix, it has been proved that the proper condition is a necessary condition for ensuring interference-free transmission [11], where the proof does not need the coefficient matrix in the polynomial equations to be generic. For the considered MIMO-IBC with i.i.d. channels, the channel matrix of each link, $\mathbf{H}_{i_k,j}$, is generic. Therefore, we can immediately extend the result in [11], i.e., the proper condition is necessary for the MIMO-IBC to be feasible. Now (3b) is proved. ■

The intuitive meaning of (3b) is to ensure that any pairs of BSs and the users in any pairs of cells should have enough spatial resources to transmit and receive their desired signals meanwhile to eliminate the ICI between these BSs and users.

Note that the necessary condition proposed in [14] was obtained by counting the total number of variables and the total number of equations in the whole network. It is only one of the proper conditions according to the definition in [10], i.e., one of the condition in (3b) when the subset \mathcal{I} is the same as the set \mathcal{J} and the subset $\mathcal{K}_i = \{1, \dots, K\}$ (i.e., it becomes the set of all users in the network).

All equations in (7) can be written in a more compact form as $\mathbf{F}(\bar{\mathbf{H}}; \bar{\mathbf{V}}, \bar{\mathbf{U}}) = \mathbf{0}$, which represents that all desired signals transmitted from the BSs are received at the users without ICI. In the sequel, we call (7) as the *ICI-free transmission equation*.

C. Proof of (3c)

Proof: To express the ICI generated from the BSs in any cells to the users in any other cells, we consider two non-overlapping clusters A and B, as shown in Fig. 1. We use \mathcal{I}_A and \mathcal{I}_B to denote

⁴For example, when $G = 3$, $\mathcal{J} = \{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$. If $\mathcal{I} = \{(1, 2)\}$, the right-hand side of (3b) is $d_2 \sum_{k \in \mathcal{K}_1} d_{1k}$, which is the number of ICIs from BS₂ to the users in \mathcal{K}_1 (an arbitrary subset of the users in cell 1), and the left-hand side is $(M_2 - d_2)d_2 + \sum_{k \in \mathcal{K}_1} (N_{1k} - d_{1k})d_{1k}$, which is the number of variables to eliminate the ICI.

the cell index sets in the clusters A and B, respectively, then $\mathcal{I}_A \cap \mathcal{I}_B = \emptyset$. Let $\mathcal{A} = \{j | j \in \mathcal{I}_A\}$ and $\mathcal{B} = \{i_k | k \in \mathcal{K}_i, i \in \mathcal{I}_B\}$ denote the BS index set in cluster A and the user index set in cluster B, respectively. The ICI-free constraints from the BSs in cluster A to the users in cluster B can be obtained from (4b) as

$$\mathbf{U}_B^H \mathbf{H}_{B,A} \mathbf{V}_A = \mathbf{0} \quad (8)$$

where $\mathbf{V}_A = \text{diag}\{\mathbf{V}_{A_1}, \dots, \mathbf{V}_{A_{|\mathcal{A}|}}\} \in \mathbb{C}^{d_A \times M_A}$, $\mathbf{U}_B = \text{diag}\{\mathbf{U}_{B_1}, \dots, \mathbf{U}_{B_{|\mathcal{B}|}}\} \in \mathbb{C}^{d_B \times N_B}$,

$$\mathbf{H}_{B,A} = \begin{bmatrix} \mathbf{H}_{B_1,A_1} & \cdots & \mathbf{H}_{B_1,A_{|\mathcal{A}|}} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{B_{|\mathcal{B}|},A_1} & \cdots & \mathbf{H}_{B_{|\mathcal{B}|},A_{|\mathcal{A}|}} \end{bmatrix} \in \mathbb{C}^{N_B \times M_A}$$

is the stacked channel matrix from the BSs in cluster A to the users in cluster B, A_s and B_s are the s th elements in \mathcal{A} and \mathcal{B} , respectively, $|\mathcal{A}| = |\mathcal{I}_A|$ and $|\mathcal{B}| = \sum_{i \in \mathcal{I}_B} |\mathcal{K}_i|$, $d_A = \sum_{j \in \mathcal{I}_A} d_j$ is the number of all data streams transmitted from the BSs in cluster A, $d_B = \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} d_{i_k}$ is the number of all data streams received at the users in cluster B, $M_A = \sum_{j \in \mathcal{I}_A} M_j$ is the number of all transmit antennas at the BSs in cluster A, and $N_B = \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} N_{i_k}$ is the number of all receive antennas at the users in cluster B.

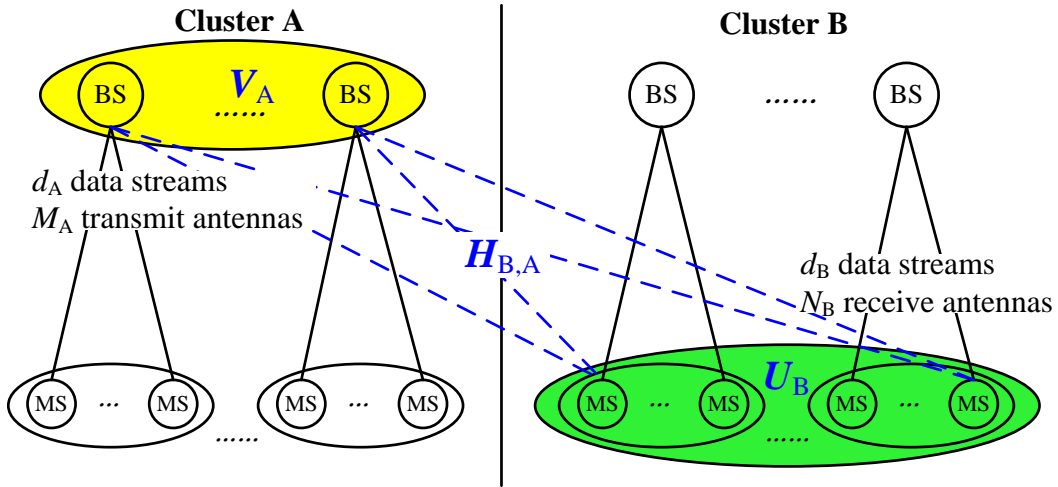


Fig. 1. ICI between arbitrary two non-overlapping clusters.

When $N_B \geq M_A$, we have $\text{rank}(\mathbf{H}_{B,A} \mathbf{V}_A) = \text{rank}(\mathbf{V}_A) = d_A$, i.e., the users in cluster B will see d_A ICIs from the BSs in cluster A. On the other hand, the users in cluster B need to receive overall d_B desired signals from the BSs in cluster B, i.e., $\text{rank}(\mathbf{U}_B) = d_B$. To separate the ICI from cluster

A and the desired signals of cluster B, the overall subspace dimension of the received signals for the users in cluster B should satisfy $N_B \geq d_A + d_B$ according to the rank-nullity theorem.

Similarly, when $N_B \leq M_A$, we have $\text{rank}(\mathbf{U}_B^H \mathbf{H}_{B,A}) = \text{rank}(\mathbf{U}_B) = d_B$, i.e., the BSs in cluster A need to avoid generating d_B ICIs to the users in cluster B. To transmit d_A desired data streams, the overall subspace dimension of the transmit signals from the BSs in cluster A should satisfy $M_A \geq d_A + d_B$.

As a result, we obtain (3c). ■

To understand the intuitive meaning of (3c), we consider a special case of it as follows,

$$M_j \geq \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} N_{i_k}, \quad \exists \mathcal{I}_B \subseteq \{1, \dots, G\} \setminus \{j\} \quad (9)$$

It indicates that there exists a BS_j in cluster A whose dimension of observation space is no less than the overall dimension of observation space at all the users in cluster B.

In the case of (9), from (3c) we have $M_j \geq d_j + \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} d_{i_k}$, which is equivalent to

$$(M_j - d_j)d_j \geq d_j \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} d_{i_k} \quad (10)$$

It means that the number of variables in the effective transmit matrix at BS_j should exceed the number of ICIs.

On the other hand, to eliminate the ICI from BS_j to the users in cluster B, (3b) ensures

$$(M_j - d_j)d_j + \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} (N_{i_k} - d_{i_k})d_{i_k} \geq d_j \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} d_{i_k} \quad (11)$$

which indicates that the overall number of variables in the transmit and receive matrices at both BS_j and the users in set \mathcal{K}_i should exceed the overall number of ICIs among them.

If a system with the configuration of (9) satisfies (11) but does not satisfy (10), it is infeasible. The infeasibility is induced by a kind of ICI, which can only be eliminated by the BS. We call them as *irreducible ICI*, because the multiple ICIs are not possible to be compressed at the BS. That is to say, only the transmit matrix at the BS is useful to avoid these ICIs, but the receive matrices at the users are not helpful to eliminate these ICIs. This implies that the ICIs between BS_j and the users in cluster B cannot be removed by their implicit “cooperation” of sharing variables in their processing matrices, i.e., these ICIs cannot be eliminated by IA.⁵

⁵When IA is employed to remove the ICI, \mathbf{U}_{i_k} and \mathbf{V}_j should work together to eliminate the ICI as shown in (4b), $i \in \mathcal{I}_B, k \in \mathcal{K}_i$.

Now we can explain the intuitive meaning of (3c). If there exists *irreducible ICI*, the BS should have enough spatial resources to avoid the ICI to these users. Otherwise, ICI-free transmission cannot be guaranteed. Correspondingly, (9) is the *condition of existence of irreducible ICI*.

By contrast, if (9) does not hold, the multiple ICIs are reducible at the BS passively, because anyway the BS only “see” these ICIs in a lower dimension. The opposite condition of (9) is the *condition of existence of reducible ICI*. In this case, both *reducible ICI* and *irreducible ICI* exist simultaneously in the network. Except for the irreducible ICI which can only be avoided by BS_j, other reducible ICIs can also be eliminated by using the spatial resources of both the BS and the users.

Similarly, for another special case where $N_{i_k} \geq \sum_{j \in \mathcal{I}_A} M_j$, $\exists \mathcal{I}_A \subseteq \{1, \dots, G\} \setminus \{i\}$, there exists *irreducible ICI*, which can only be canceled by MS_{i_k}.

IV. NECESSARY AND SUFFICIENT CONDITIONS

In this section, we present and prove *necessary and sufficient conditions* for the feasibility of interference-free transmission in a special class of MIMO-IBC. Owing to the second feature of MIMO-IBC, the proof of the sufficient conditions for MIMO-IBC is more difficult than the special class of MIMO-IC in [11].

After we prove the necessary condition, we will analyze the similarity and difference of MIMO-IBC from multi-beam MIMO-IC in the proof of the sufficient conditions. From the analysis we will show when the proof for MIMO-IC can be extended to MIMO-IBC, when cannot and what is the difficulty. Then, we will prove the sufficient conditions for the MIMO-IBC whose proof cannot be extended from MIMO-IC.

Theorem 2 (Necessary and Sufficient Conditions): In a special class of MIMO-IBC with configuration $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, d))$ where the channel matrix in each link is generic, when both M_i and N_{i_k} are divisible by d , the interference-free transmission is feasible iff (if and only if) the following conditions are satisfied,

$$\min\{M_i - K_i d, N_{i_k} - d\} \geq 0, \forall i, k \quad (12a)$$

$$\sum_{j:(i,j) \in \mathcal{I}} (M_j - K_j d) K_j + \sum_{i:(i,j) \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} (N_{i_k} - d) \geq \sum_{(i,j) \in \mathcal{I}} K_j |\mathcal{K}_i| d, \forall \mathcal{I} \subseteq \mathcal{J} \quad (12b)$$

A. Proof of the necessary conditions

Proof: Comparing *Theorem 1* and *Theorem 2*, we can see that (12a) and (12b) are the reduced form of (3a) and (3b) for the special class of MIMO-IBC. For the considered MIMO-IBC, (3c) becomes

$$\max \left\{ \sum_{j \in \mathcal{I}_A} M_j, \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} N_{i_k} \right\} \geq \sum_{j \in \mathcal{I}_A} K_j d + \sum_{i \in \mathcal{I}_B} |\mathcal{K}_i| d \quad (13)$$

In the sequel, we show that (13) can be derived from (12a) and (12b).

Since M_j is integral multiples of d , the value of M_j can be divided into two cases:

- 1) $\sum_{j \in \mathcal{I}_A} M_j \geq (\sum_{j \in \mathcal{I}_A} K_j + \sum_{i \in \mathcal{I}_B} |\mathcal{K}_i|)d$,
- 2) $\sum_{j \in \mathcal{I}_A} M_j \leq (\sum_{j \in \mathcal{I}_A} K_j + \sum_{i \in \mathcal{I}_B} |\mathcal{K}_i| - 1)d$.

In the first case, (13) always holds. In the second case, we have

$$\sum_{i \in \mathcal{I}_B} |\mathcal{K}_i| d - \sum_{j \in \mathcal{I}_A} (M_j - K_j d) \geq d \quad (14)$$

Considering (12a), we know that $M_j - K_j d \geq 0$. Thus the inequality $\sum_{j \in \mathcal{I}_A} (M_j - K_j d) \geq M_j - K_j d$ always holds. Substituting this inequality into (14), we have

$$\sum_{i \in \mathcal{I}_B} |\mathcal{K}_i| d - (M_j - K_j d) \geq d \quad (15)$$

Considering the definition of \mathcal{I}_A and \mathcal{I}_B in (13), (12b) can be rewritten as $\sum_{j \in \mathcal{I}_A} (M_j - K_j d) K_j + \sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} (N_{i_k} - d) \geq \sum_{j \in \mathcal{I}_A} K_j \sum_{i \in \mathcal{I}_B} |\mathcal{K}_i| d$, which is equivalent to

$$\sum_{i \in \mathcal{I}_B} \sum_{k \in \mathcal{K}_i} N_{i_k} \geq \sum_{j \in \mathcal{I}_A} K_j \left(\sum_{i \in \mathcal{I}_B} |\mathcal{K}_i| d - (M_j - K_j d) \right) + \sum_{i \in \mathcal{I}_B} |\mathcal{K}_i| d \quad (16)$$

Substituting (15) into (16), we obtain (13). Consequently, (13) is not necessary. ■

In the considered class of MIMO-IBC, since (13) (i.e., (3c)) can be derived from (12b) (i.e., (3b)), the proper condition ensures that when there exists *irreducible ICI*, the BS or user has enough spatial resources to avoid (or cancel) the ICI.

B. Proof of the sufficient conditions

1) *Similarities and differences with MIMO-IC:* Implicit function theorem provides a general way to prove the solvability of equations. The theorem ensures that, if there exists a channel matrix $\bar{\mathbf{H}}_0$ and transmit and receive matrices $\{\bar{\mathbf{V}}_0, \bar{\mathbf{U}}_0\}$ that satisfies the ICI-free transmission equation in (7) and the corresponding Jacobian matrix is invertible, there will be a mapping from the space of

\bar{H}_0 to the space of $\{\bar{V}_0, \bar{U}_0\}$ and the mapping is locally full-dimensional (i.e., full-dimensional in the neighborhood of $(\bar{H}_0; \bar{V}_0, \bar{U}_0)$) [11]. The Jacobian matrix of (7) is defined as

$$\mathbf{J} \triangleq \left[\frac{\partial \text{vec}\{\mathbf{F}\}}{\partial \text{vec}\{\bar{\mathbf{V}}; \bar{\mathbf{U}}\}} \right] = \left[\frac{\partial \text{vec}\{\mathbf{F}\}}{\partial \text{vec}\{\bar{\mathbf{V}}\}}, \frac{\partial \text{vec}\{\mathbf{F}\}}{\partial \text{vec}\{\bar{\mathbf{U}}\}} \right] = [\mathbf{J}^V, \mathbf{J}^U] \quad (17)$$

where $\text{vec}\{\bar{\mathbf{V}}\} = [\text{vec}\{\bar{\mathbf{V}}_{11}\}^T, \dots, \text{vec}\{\bar{\mathbf{V}}_{G_{K_G}}\}^T]^T$, and $\text{vec}\{\bar{\mathbf{U}}\} = [\text{vec}\{\bar{\mathbf{U}}_{11}^*\}^T, \dots, \text{vec}\{\bar{\mathbf{U}}_{G_{K_G}}^*\}^T]^T$.

Since the image of the mapping is locally full-dimensional, the authors in [11] proved that the ICI-free transmission equation is solvable by using Chevalley's theorem [16, Chapter 2,6.E.]. That is to say, a key step in proving the sufficient conditions is to find $(\bar{H}_0; \bar{V}_0, \bar{U}_0)$ that simultaneously satisfies the following two conditions: 1) $\mathbf{F}(\bar{H}_0; \bar{V}_0, \bar{U}_0) = \mathbf{0}$, 2) $\mathbf{J}(\bar{H}_0; \bar{V}_0, \bar{U}_0)$ is nonsingular.

From the proof in [11], we can summarize three important observations as follows, which is useful to simplify the way to construct the invertible Jacobian matrix for MIMO-IBC.

Observation 1: The above two conditions can be decoupled by setting $\bar{H}_{0\ i_k,j}^{(1)} = \mathbf{0}$ and $\bar{H}_{0\ i_k,j}^{(4)} = \mathbf{0}$.

This is because when $\bar{H}_{0\ i_k,j}^{(1)} = \mathbf{0}$ and $\bar{H}_{0\ i_k,j}^{(4)} = \mathbf{0}$, the equation $\mathbf{F}(\bar{H}_0; \bar{V}_0, \bar{U}_0) = \mathbf{0}$ always has a zero solution, i.e., $\bar{V}_0 = \mathbf{0}$ and $\bar{U}_0 = \mathbf{0}$. Therefore, the equation is independent of $\bar{H}_{0\ i_k,j}^{(2)}$ and $\bar{H}_{0\ i_k,j}^{(3)}$, while the Jacobian matrix only depends on $\bar{H}_{0\ i_k,j}^{(2)}$ and $\bar{H}_{0\ i_k,j}^{(3)}$. Hence, we only need to find $\bar{H}_{0\ i_k,j}^{(2)}$ and $\bar{H}_{0\ i_k,j}^{(3)}$ to make Jacobian matrix invertible.

Based on this observation, in the sequel we will only consider the case of $\bar{H}_{0\ i_k,j}^{(1)} = \mathbf{0}$ and $\bar{H}_{0\ i_k,j}^{(4)} = \mathbf{0}$. Then, $\mathbf{J}(\bar{H}_0; \bar{V}_0, \bar{U}_0)$ does not depend on \bar{V}_0 and \bar{U}_0 , i.e., $\mathbf{J}(\bar{H}_0; \bar{V}_0, \bar{U}_0) = \mathbf{J}(\bar{H}_0)$. Since $\mathbf{F}(\bar{H}; \bar{V}, \bar{U})$ is continuously differentiable, we have

$$\mathbf{J}(\bar{H}_0) = \left[\frac{\partial \text{vec}\{\mathbf{F}\}}{\partial \text{vec}\{\bar{\mathbf{V}}; \bar{\mathbf{U}}\}} \right] \Big|_{\bar{H}=\bar{H}_0} = \left[\frac{\partial \text{vec}\{\mathbf{F}(\bar{H}_0)\}}{\partial \text{vec}\{\bar{\mathbf{V}}; \bar{\mathbf{U}}\}} \right] \quad (18)$$

where $\mathbf{F}_{i_k,j}(\bar{H}_0) \triangleq \mathbf{F}_{i_k,j}(\bar{H}; \bar{V}, \bar{U})|_{\bar{H}=\bar{H}_0}$, and from (7) we obtain

$$\mathbf{F}_{i_k,j}(\bar{H}_0) = \bar{H}_{0\ i_k,j}^{(2)} \bar{V}_j + \bar{U}_{i_k}^H \bar{H}_{0\ i_k,j}^{(3)} = \mathbf{0}, \quad \forall i \neq j \quad (19)$$

To see the structure of Jacobian matrix of MIMO-IBC, we rewrite the matrices for the users in each cell as $\mathbf{F}_{i_k,j} = [\mathbf{F}_{i_k,j_1}, \dots, \mathbf{F}_{i_k,j_{K_j}}]$, $\bar{\mathbf{V}}_j = [\bar{\mathbf{V}}_{j_1}, \dots, \bar{\mathbf{V}}_{j_{K_j}}]$, and $\bar{H}_{0\ i_k,j}^{(3)} = [\bar{H}_{0\ i_k,j_1}^{(3)}, \dots, \bar{H}_{0\ i_k,j_{K_j}}^{(3)}]$, where $\mathbf{F}_{i_k,j_l} \in \mathbb{C}^{d \times d}$, $\bar{\mathbf{V}}_{j_l} \in \mathbb{C}^{(M_j - K_j d) \times d}$, and $\bar{H}_{0\ i_k,j_l}^{(3)} \in \mathbb{C}^{(N_{i_k} - d) \times d}$. Substituting them into (19), we obtain K_j groups of ICI-free transmission subequations of MIMO-IBC for cell j , which are

$$\begin{cases} \mathbf{F}_{i_k,j_1}(\bar{H}_0) = \bar{H}_{0\ i_k,j}^{(2)} \bar{\mathbf{V}}_{j_1} + \bar{U}_{i_k}^H \bar{H}_{0\ i_k,j_1}^{(3)} = \mathbf{0} \\ \dots \\ \mathbf{F}_{i_k,j_{K_j}}(\bar{H}_0) = \bar{H}_{0\ i_k,j}^{(2)} \bar{\mathbf{V}}_{j_{K_j}} + \bar{U}_{i_k}^H \bar{H}_{0\ i_k,j_{K_j}}^{(3)} = \mathbf{0} \end{cases} \quad (20)$$

where $\mathbf{F}_{i_k, j_l}(\cdot)$ represents the ICI from $\bar{\mathbf{V}}_{j_l}$ to $\bar{\mathbf{U}}_{i_k}$, i.e., the interference generated by the effective transmit matrix for the l th user of BS $_j$ to the k th user in cell i , $l = 1, \dots, K_j$.

Since M_j and N_{i_k} are divisible by d , the channel, transmit and receive matrices can be partitioned into blocks of size $d \times d$ and the l th subequation in (20) can be further rewritten as

$$\begin{aligned} \mathbf{F}_{i_k, j_l}(\bar{\mathbf{H}}_0) &= \bar{\mathbf{H}}_{0 \ i_k, j}^{(2)} \bar{\mathbf{V}}_{j_l} + \bar{\mathbf{U}}_{i_k}^H \bar{\mathbf{H}}_{0 \ i_k, j_l}^{(3)} \\ &= \sum_{t=1}^{M_j/d-K_j} \bar{\mathbf{H}}_{i_k, j}^{(2), t} \bar{\mathbf{V}}_{j_l(t)} + \sum_{s=1}^{N_{i_k}/d-1} \bar{\mathbf{U}}_{i_k(s)}^H \bar{\mathbf{H}}_{i_k, j_l}^{(3), s} = \mathbf{0}, \quad \forall i \neq j \end{aligned} \quad (21)$$

where $\bar{\mathbf{V}}_{j_l(t)}$ and $\bar{\mathbf{H}}_{0 \ i_k, j}^{(2), t}$ are t th block of size $d \times d$ in $\bar{\mathbf{V}}_{j_l}$ and $\bar{\mathbf{H}}_{0 \ i_k, j}^{(2)}$, $t = 1, \dots, M_j/d - K_j$, $\bar{\mathbf{U}}_{i_k(s)}$ and $\bar{\mathbf{H}}_{0 \ i_k, j_l}^{(3), s}$ are s th block of size $d \times d$ in $\bar{\mathbf{U}}_{i_k}$ and $\bar{\mathbf{H}}_{0 \ i_k, j_l}^{(3)}$, $s = 1, \dots, N_{i_k}/d - 1$.

The elements of the Jacobian matrix can be obtained by taking partial derivatives over (21), which are

$$\frac{\partial \text{vec}\{\mathbf{F}_{i_k, j_l}(\bar{\mathbf{H}}_0)\}}{\partial \text{vec}\{\bar{\mathbf{V}}_{m_n(t)}\}} = \begin{cases} \bar{\mathbf{H}}_{0 \ i_k, j}^{(2), t} \otimes \mathbf{I}_d, & \forall m_n = j_l \\ \mathbf{0}_{d^2}, & \forall m_n \neq j_l \end{cases} \quad (22a)$$

$$\frac{\partial \text{vec}\{\mathbf{F}_{i_k, j_l}(\bar{\mathbf{H}}_0)\}}{\partial \text{vec}\{\bar{\mathbf{U}}_{m_n(s)}^*\}} = \begin{cases} \mathbf{I}_d \otimes (\bar{\mathbf{H}}_{0 \ i_k, j_l}^{(3), s})^T, & \forall m_n = i_k \\ \mathbf{0}_{d^2}, & \forall m_n \neq i_k \end{cases} \quad (22b)$$

where $t = 1, \dots, M_j/d - K_j$, $s = 1, \dots, N_{i_k}/d - 1$.

Observation 2: When each user has d data streams, and M_j and N_{i_k} are divisible by d , an invertible Jacobian matrix for the case of $d > 1$ can be constructed from that for $d = 1$.

In (22a) and (22b), if let $\bar{\mathbf{H}}_{0 \ i_k, j}^{(2), t} = \bar{h}_{0 \ i_k, j}^{(2), t} \mathbf{I}_d$ and $\bar{\mathbf{H}}_{0 \ i_k, j_l}^{(3), s} = \bar{h}_{0 \ i_k, j_l}^{(3), s} \mathbf{I}_d$, where $\bar{h}_{0 \ i_k, j}^{(2), t}$ and $\bar{h}_{0 \ i_k, j_l}^{(3), s}$ are the $(1, 1)$ th elements of $\bar{\mathbf{H}}_{0 \ i_k, j}^{(2), t}$ and $\bar{\mathbf{H}}_{0 \ i_k, j_l}^{(3), s}$, respectively, we have $\bar{\mathbf{H}}_{0 \ i_k, j}^{(2), t} \otimes \mathbf{I}_d = \bar{h}_{0 \ i_k, j}^{(2), t} \mathbf{I}_{d^2}$ and $\mathbf{I}_d \otimes \bar{\mathbf{H}}_{0 \ i_k, j_l}^{(3), s} = \bar{h}_{0 \ i_k, j_l}^{(3), s} \mathbf{I}_{d^2}$. As a result, the Jacobian matrix for a $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, d))$ MIMO-IBC can be rewritten as $\mathbf{J}(\bar{\mathbf{H}}_0) = \tilde{\mathbf{J}}(\bar{\mathbf{H}}_0) \otimes \mathbf{I}_{d^2}$, where $\tilde{\mathbf{J}}(\bar{\mathbf{H}}_0)$ has the same pattern of nonzero elements as the Jacobian matrix for a $\prod_{i=1}^G (M_i/d \times \prod_{k=1}^{K_i} (N_{i_k}/d, 1))$ MIMO-IBC. Therefore, once we construct an invertible matrix $\tilde{\mathbf{J}}(\bar{\mathbf{H}}_0)$, we can obtain an invertible matrix $\mathbf{J}(\bar{\mathbf{H}}_0)$ immediately.

Based on this observation, we only need to investigate the case of $d = 1$. Then the effective transmit and receive matrices $\bar{\mathbf{V}}_{j_l}$ and $\bar{\mathbf{U}}_{i_k}$ in (19) reduce to transmit and receive vectors $\bar{\mathbf{v}}_{j_l}$ and

$\bar{\mathbf{u}}_{i_k}$. Therefore, the ICI-free transmission subequation in (21) is simplified as

$$\begin{aligned} F_{i_k, j_l}(\bar{\mathbf{H}}_0) &= \bar{\mathbf{h}}_{0, i_k, j}^{(2)} \bar{\mathbf{v}}_{j_l} + \bar{\mathbf{u}}_{i_k}^H \bar{\mathbf{h}}_{0, i_k, j_l}^{(3)} \\ &= \sum_{t=1}^{M_j - K_j} \bar{h}_{0, i_k, j}^{(2), t} \bar{v}_{j_l(t)} + \sum_{s=1}^{N_{i_k} - 1} \bar{u}_{i_k(s)}^* \bar{h}_{0, i_k, j_l}^{(3), s} = 0, \quad \forall i \neq j \end{aligned} \quad (23)$$

where $\bar{v}_{i_k(t)}$ and $\bar{h}_{0, i_k, j}^{(2), t}$ are the t th element of $\bar{\mathbf{v}}_{j_l}$ and $\bar{\mathbf{h}}_{0, i_k, j}^{(2)}$, $\bar{u}_{i_k(s)}$ and $\bar{h}_{0, i_k, j_l}^{(3), s}$ are the s th element of $\bar{\mathbf{u}}_{i_k}$ and $\bar{\mathbf{h}}_{0, i_k, j_l}^{(3)}$.

The elements of the Jacobian matrix now become

$$\frac{\partial F_{i_k, j_l}(\bar{\mathbf{H}}_0)}{\partial \bar{v}_{m_n(t)}} = \begin{cases} \bar{h}_{0, i_k, j}^{(2), t}, & \forall m_n = j_l \\ 0, & \forall m_n \neq j_l \end{cases} \quad (24a)$$

$$\frac{\partial F_{i_k, j_l}(\bar{\mathbf{H}}_0)}{\partial \bar{u}_{m_n(s)}^*} = \begin{cases} \bar{h}_{0, i_k, j_l}^{(3), s}, & \forall m_n = i_k \\ 0, & \forall m_n \neq i_k \end{cases} \quad (24b)$$

where $t = 1, \dots, M_j - K_j$ and $s = 1, \dots, N_{i_k} - 1$.

In (24a) and (24b), the nonzero elements satisfy

$$\frac{\partial F_{i_k, j_l}(\bar{\mathbf{H}}_0)}{\partial \bar{v}_{j_l(t)}} = \dots = \frac{\partial F_{i_k, j_{K_j}}(\bar{\mathbf{H}}_0)}{\partial \bar{v}_{j_{K_j}(t)}} = \bar{h}_{0, i_k, j}^{(2), t} \quad (25a)$$

$$\frac{\partial F_{i_1, j_l}(\bar{\mathbf{H}}_0)}{\partial \bar{u}_{i_1(s)}^*} = \bar{h}_{0, i_1, j_l}^{(3), s}, \quad \dots, \quad \frac{\partial F_{i_{K_i}, j_l}(\bar{\mathbf{H}}_0)}{\partial \bar{u}_{i_{K_i}(s)}^*} = \bar{h}_{0, i_{K_i}, j_l}^{(3), s} \quad (25b)$$

It is shown from (25a) that the elements of Jacobian matrix of the MIMO-IBC are not generic any more (they are repeated) when $d = 1$, which is different from the single beam MIMO-IC.

Observation 3: The invertible Jacobian matrix in the case of $L_v > L_e$ can be obtained from that in the case of $L_v = L_e$, where L_v and L_e denote the total number of scalar variables and equations in (23), respectively.

When $L_v > L_e$, i.e., there are more variables than the equations, one can always remove some redundant variables to ensure $L_v = L_e$.

According to these observations, we only need to investigate the invertibility of Jacobian matrix of (23) for the case where $d = 1$, $\bar{\mathbf{h}}_{0, i_k, j_l}^{(1)} = \mathbf{0}$, $\bar{\mathbf{h}}_{0, i_k, j_l}^{(4)} = \mathbf{0}$ and $L_v = L_e$. In this case, $L_v = \sum_{j=1}^G (M_j - K_j) K_j + \sum_{i=1}^G \sum_{k=1}^{K_i} (N_{i_k} - 1)$ and $L_e = \sum_{i=1}^G \sum_{j=1, j \neq i}^G K_i K_j$. Considering (3a) and $L_v = L_e$, it is not difficult to derive that M_j and N_{i_k} need to satisfy

$$\sum_{i=1, i \neq j}^G d_i \geq M_j - d_j \geq 0, \quad \sum_{j=1, j \neq i}^G d_j \geq N_i - d_{i_k} \geq 0 \quad (26)$$

In this case, when $\text{rank}(\mathbf{J}) = \sum_{i=1}^G \sum_{j=1, j \neq i}^G K_i K_j$, the Jacobian matrix of (23) is invertible.

In the following, we provide two lemmas for two special cases of the MIMO-IBC in *Theorem 2*. We will show that the sufficiency proof for which kinds of MIMO-IBC can and which cannot be extended from the proof for MIMO-IC in [11].

Lemma 1: (Similarity with MIMO-IC) For a MIMO-IBC $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, 1))$, when $N_{i_1} = \dots = N_{i_{K_i}}, K_1 = \dots = K_G = K$, and both M_i and $N_{i_k} - 1$ are divisible by K , interference-free transmission is feasible iff (12a) and (12b) are satisfied. The corresponding invertible Jacobian matrix can be constructed using the similar way as in [11].

Proof: To construct an invertible Jacobian matrix for a $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, 1))$ MIMO-IBC, where BS_j supports K_j users and each user only receives one data stream, we first consider a $\prod_{i=1}^G (M_i \times N_i, K_i)$ MIMO-IC, where BS_j only supports one user and the user in cell j receives K_j data streams. For concise, we use $\mathbf{J}_{\text{IBC}}(\bar{\mathbf{H}}_0)$ and $\mathbf{J}_{\text{IC}}(\bar{\mathbf{H}}_0)$ to denote the Jacobian matrix of the $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, 1))$ MIMO-IBC and that of the $\prod_{i=1}^G (M_i \times N_i, K_i)$ MIMO-IC, respectively.

For the $\prod_{i=1}^G (M_i \times N_i, K_i)$ MIMO-IC, to ensure the l th data stream transmitted from BS_j not to interfere the k th data stream received at MS_i , the ICI-free transmission equation is

$$\begin{aligned} F_{i_k, j_l}^{\text{IC}}(\bar{\mathbf{H}}_0) &= \bar{\mathbf{h}}_{0, i_k, j}^{(2)} \bar{\mathbf{v}}_{j_l} + \bar{\mathbf{u}}_{i_k}^H \bar{\mathbf{h}}_{0, i, j_l}^{(3)} \\ &= \sum_{t=1}^{M_j - K_j} \bar{h}_{0, i_k, j}^{(2), t} \bar{v}_{j_l(t)} + \sum_{s=1}^{N_i - K_i} \bar{u}_{i_k(s)}^* \bar{h}_{0, i, j_l}^{(3), s} = 0, \quad \forall i \neq j \end{aligned} \quad (27)$$

where $\bar{v}_{j_l(s)}$ and $\bar{h}_{0, i, j_l}^{(2), t}$ are the t th elements of $\bar{\mathbf{v}}_{j_l}$ and $\bar{\mathbf{h}}_{0, i_k, j}^{(2)}$, $\bar{u}_{i_k(s)}$ and $\bar{h}_{0, i, j_l}^{(3), s}$ are the s th elements of $\bar{\mathbf{u}}_{i_k}$ and $\bar{\mathbf{h}}_{0, i, j_l}^{(3)}$. From (27) we can obtain the elements of Jacobian matrix as follows,

$$\frac{\partial F_{i_k, j_l}^{\text{IC}}(\bar{\mathbf{H}}_0)}{\partial \bar{v}_{m_n(t)}} = \begin{cases} \bar{h}_{0, i_k, j}^{(2), t}, & \forall m_n = j_l \\ 0, & \forall m_n \neq j_l \end{cases} \quad (28a)$$

$$\frac{\partial F_{i_k, j_l}^{\text{IC}}(\bar{\mathbf{H}}_0)}{\partial \bar{u}_{m_n(s)}^*} = \begin{cases} \bar{h}_{0, i, j_l}^{(3), s}, & \forall m_n = i_k \\ 0, & \forall m_n \neq i_k \end{cases} \quad (28b)$$

where $t = 1, \dots, M_j - K_j$ and $s = 1, \dots, N_i - K_i$.

In (28a) and (28b), the nonzero elements satisfy

$$\frac{\partial F_{i_k, j_l}^{\text{IC}}(\bar{\mathbf{H}}_0)}{\partial \bar{v}_{j_l(t)}} = \dots = \frac{\partial F_{i_k, j_{K_j}}^{\text{IC}}(\bar{\mathbf{H}}_0)}{\partial \bar{v}_{j_{K_j}(t)}} = \bar{h}_{0, i_k, j}^{(2), t} \quad (29a)$$

$$\frac{\partial F_{i_1, j_l}^{\text{IC}}(\bar{\mathbf{H}}_0)}{\partial \bar{u}_{i_1(s)}^*} = \dots = \frac{\partial F_{i_{K_i}, j_l}^{\text{IC}}(\bar{\mathbf{H}}_0)}{\partial \bar{u}_{i_{K_i}(s)}^*} = \bar{h}_{0, i, j_l}^{(3), s} \quad (29b)$$

Comparing (22a) and (22b) with (28a) and (28b), it is easy to find that when $N_{i_k} - 1 = N_i - K_i$, $\mathbf{J}_{\text{IBC}}(\bar{\mathbf{H}}_0)$ has the same nonzero entry pattern with $\mathbf{J}_{\text{IC}}(\bar{\mathbf{H}}_0)$.

Moreover, comparing (25a) and (29a), we can see that the repeated nonzero elements in $\mathbf{J}_{\text{IBC}}^V(\bar{\mathbf{H}}_0)$ have the same pattern as those in $\mathbf{J}_{\text{IC}}^V(\bar{\mathbf{H}}_0)$. By contrast, comparing (25b) and (29b), we can see that the nonzero elements of $\mathbf{J}_{\text{IBC}}^U(\bar{\mathbf{H}}_0)$ are generic but those of $\mathbf{J}_{\text{IC}}^U(\bar{\mathbf{H}}_0)$ are not since they are repeated. This suggests that the elements of $\mathbf{J}_{\text{IBC}}(\bar{\mathbf{H}}_0)$ are more flexible to be set into any value than that of $\mathbf{J}_{\text{IC}}(\bar{\mathbf{H}}_0)$. Hence, if there exists an invertible $\mathbf{J}_{\text{IC}}(\bar{\mathbf{H}}_0)$, we can obtain an invertible $\mathbf{J}_{\text{IBC}}(\bar{\mathbf{H}}_0)$ by setting $\mathbf{J}_{\text{IBC}}(\bar{\mathbf{H}}_0) = \mathbf{J}_{\text{IC}}(\bar{\mathbf{H}}_0)$.

For the $\prod_{i=1}^G (M_i \times N_i, K_i)$ MIMO-IC, it was proved in [11] that a proper system is feasible when each user transmits the same number of data streams, i.e., $K_1 = \dots = K_G = K$, and M_i and N_i are divisible by K . From above comparison for the Jacobian matrices of the MIMO-IBC and MIMO-IC, we know that for the $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, 1))$ MIMO-IBC: a proper system is feasible when each cell supports the same number of users, and both M_i and $N_{i_k} - 1$ are divisible by K . ■

Lemma 2: (Difference with MIMO-IC) For a MIMO-IBC $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, 1))$ where $L_v = L_e$, when $\sum_{j=1, j \neq i}^G d_j \geq N_{i_k} - 1 > 0$ and $N_{i_k} - 1 \notin \phi_i, \exists i, k$, where $\phi_i = \{\sum_{j \in \psi_i} K_j | \psi_i \subseteq \{1, \dots, G\} \setminus \{i\}\}$ ⁶, interference-free transmission is feasible iff (12a) and (12b) are satisfied. However, the corresponding invertible Jacobian matrix cannot be constructed with similar method in [11].

Before proving *Lemma 2*, we first briefly review the method to construct the invertible Jacobian matrix in [11], where the graph theory was applied. Similar idea was introduced in [17] to construct a nonsingular Jacobian matrix. From [11], [17], we know that the ICI-free transmission equation can be represented by a bipartite graph. The bipartite graph can be expressed by an adjacency matrix \mathbf{D} . It represents which vertices in one set of a graph are connected to the vertices in the other set. In [17], the authors revealed the relationship between the adjacency matrix and the Jacobian matrix: they have the same pattern of nonzero entries. This suggests that a nonsingular $\mathbf{J}(\bar{\mathbf{H}}_0)$ can be constructed from a nonsingular \mathbf{D} . In graph theory, a set of nonadjacent edges in a graph is called a matching. If a matching matches all vertices of the graph, it is called a *perfect matching*,

⁶Here, ϕ_i is a set of user number in one or multiple cells, whose desired signals will cause ICI to the users in cell i . For example, when $G = 3$, $\phi_1 = \{K_2, K_3, K_2 + K_3\}$, $\phi_2 = \{K_1, K_3, K_1 + K_3\}$ and $\phi_3 = \{K_1, K_2, K_1 + K_2\}$. When $K_1 = \dots = K_G = K$, $N_{i_k} - 1 \notin \phi_i$ reduces to the case that $N_{i_k} - 1$ is not divisible by K , which is just opposite to the case of *Lemma 1*.

i.e., the two sets of vertices have a one-to-one mapping relationship. For a perfect matching in the bipartite graph, denote the corresponding adjacency matrix as $\mathbf{D}^{\mathcal{W}}$. According to the definition of perfect matching, we know that $\mathbf{D}^{\mathcal{W}}$ is a permutation matrix. In a MIMO-IC where $d = 1$, the nonzero elements of $\mathbf{J}(\bar{\mathbf{H}}_0)$ are generic, therefore a nonsingular Jacobian matrix can be obtained by setting $\mathbf{J}(\bar{\mathbf{H}}_0) = \mathbf{D}^{\mathcal{W}}$. From Hall's theorem [18, Theorem 3.1.11] who provides the existence condition of perfect matching, which happens to be the same as the proper condition, we know that when a system is proper there exists a perfect matching in the bipartite graph. Therefore, the proper MIMO-IC system with $d = 1$ is feasible.

Proof: Note that the MIMO-IBC considered in Lemma 2 is a special class of MIMO-IBC in Theorem 2. Therefore, as we will prove soon, interference-free transmission is feasible in this class of MIMO-IBC iff (12a) and (12b) are satisfied.

In the following, we prove that the corresponding invertible Jacobian matrix cannot be constructed with similar method as in [11] by contradiction. To this end, we assume that the method in [11], [17] can be applied to construct the nonsingular Jacobian matrix for the considered MIMO-IBC. We represent (23) as a bipartite graph, $C = (\mathcal{X}, \mathcal{Y}, \mathcal{E})$, where $\mathcal{Y} = \{F_{i_k, j_l} | k = 1, \dots, K_i, l = 1, \dots, K_j, \forall i \neq j\}$ is the set of vertices representing the ICI-free transmission equations, $\mathcal{X}^V = \{\bar{v}_{j_l(t)} | t = 1, \dots, M_j - K_j, l = 1, \dots, K_j, j = 1, \dots, G\}$, $\mathcal{X}^U = \{\bar{u}_{i_k(s)}^* | s = 1, \dots, N_{i_k} - 1, k = 1, \dots, K_i, i = 1, \dots, G\}$, $\mathcal{X} = \mathcal{X}^V \cup \mathcal{X}^U$ is the set of vertices representing the scalar variables in (23), \mathcal{E} is the set of edges and $[Y_m, X_n] \in \mathcal{E}$ iff equation Y_m contains variable X_n , where X_m and Y_m are the m th entry in \mathcal{X} and \mathcal{Y} , respectively.

Let \mathbf{D} denote the adjacency matrix of the bipartite graph, whose rows correspond to \mathcal{Y} and columns correspond to \mathcal{X} and whose (m, n) th element is

$$D_{m,n} = \begin{cases} 1, & \forall [Y_m, X_n] \in \mathcal{E} \\ 0, & \text{otherwise.} \end{cases} \quad (30)$$

Hall's theorem indicates that in a bipartite graph $C = (\mathcal{X}, \mathcal{Y}, \mathcal{E})$, for a set \mathcal{S} , a perfect matching exists iff $|N(\mathcal{S})| \geq |\mathcal{S}|$, $\forall \mathcal{S} \subseteq \mathcal{Y}$, where $N(\mathcal{S})$ is the set of all vertices adjacent to some elements of \mathcal{S} .

In the considered MIMO-IBC, for any given set $\mathcal{S} \subseteq \mathcal{Y}$, there exist corresponding sets $\mathcal{I} \subseteq \mathcal{J}$ and $\mathcal{K}_i \subseteq \{1, \dots, K_i\}$. Since $|\mathcal{S}| = \sum_{(i,j) \in \mathcal{I}} K_j |\mathcal{K}_i|$ and $|N(\mathcal{S})| = \sum_{j:(i,j) \in \mathcal{I}} (M_j - K_j) K_j + \sum_{i:(i,j) \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} (N_{i_k} - 1)$, $|N(\mathcal{S})| \geq |\mathcal{S}|$ is actually the proper condition for the $\prod_{i=1}^G (M_i \times$

$\prod_{k=1}^{K_i} (N_{i_k}, 1)$) MIMO-IBC. Therefore, according to Hall's theorem we know that when the MIMO-IBC is proper, there exists a perfect matching. In the perfect matching, each variable in \mathcal{X} is assigned to deal with one and the only one ICI in \mathcal{Y} , thus the elements of the corresponding adjacency matrix $D^{\mathcal{W}}$ are

$$D_{m,n}^{\mathcal{W}} = \begin{cases} 1, & \forall \mathcal{X}_n \text{ corresponds } \mathcal{Y}_m \\ 0, & \text{otherwise.} \end{cases} \quad (31)$$

who satisfy $\sum_{m=1}^{L_e} D_{m,n}^{\mathcal{W}} = 1$ and $\sum_{n=1}^{L_v} D_{m,n}^{\mathcal{W}} = 1$.

From (23) and considering $C = (\mathcal{X}, \mathcal{Y}, \mathcal{E})$, the element of the Jacobian matrix for the MIMO-IBC is

$$J_{m,n} = \frac{\partial Y_m}{\partial X_n} = \begin{cases} \neq 0, & \forall [Y_m, X_n] \in \mathcal{E} \\ 0, & \text{otherwise.} \end{cases} \quad (32)$$

Comparing (30) and (32), we see that $J(\bar{H}_0)$ has the same pattern of nonzero entries as D . Therefore, we can construct an invertible Jacobian matrix from $D^{\mathcal{W}}$.

If the effective transmit variable $\bar{v}_{j_l(t)}$ is assigned to avoid the ICI F_{i_k, j_l} , setting $J(\bar{H}_0) = D^{\mathcal{W}}$ requires $\bar{h}_{0, i_k, j}^{(2), t} = 1$, otherwise $\bar{h}_{0, i_k, j}^{(2), t} = 0$. On the other hand, the nonzero elements of $J(\bar{H}_0)$ are repeated as shown in (25a), i.e., $\partial F_{i_k, j_1}(\bar{H}_0) / \partial \bar{v}_{j_1(t)} = \dots = \partial F_{i_k, j_{K_j}}(\bar{H}_0) / \partial \bar{v}_{j_{K_j}(t)} = \bar{h}_{0, i_k, j}^{(2), t}$. This implies that once a transmit variable $\bar{v}_{j_1(t)}$ at BS_{*j*} is assigned to avoid the ICI F_{i_k, j_1} , other transmit variables $\bar{v}_{j_l(t)}$ at BS_{*j*} should be assigned to avoid the ICIs F_{i_k, j_l} , $l = 2, \dots, K_j$. That is to say, BS_{*j*} needs to avoid all the ICI it generated to MS_{*i_k*}.

For MS_{*i_k*}, the number of ICIs it experienced is an element in ϕ_i , and the number of variables in its effective receive vector to remove these ICIs is $N_{i_k} - 1$. When $N_{i_k} - 1 \leq \sum_{j=1, j \neq i}^G d_j$ and $N_{i_k} - 1 \notin \phi_i$, there will exist one BS (say BS_{*j*}) where the number of variables at MS_{*i_k*} is not large enough to cancel all the ICIs generated from BS_{*j*}. When $N_{i_k} - 1 > 0$, MS_{*i_k*} is able to cancel a part of ICI from BS_{*j*}, which means BS_{*j*} can not avoid all the ICIs to MS_{*i_k*} considering $L_v = L_e$. Consequently, the conditions of $N_{i_k} - 1$ imply that the ICIs from BS_{*j*} to MS_{*i_k*}, $F_{i_k, j_1}, \dots, F_{i_k, j_{K_j}}$, need to be eliminated by BS_{*j*} and MS_{*i_k*} jointly.

Since BS_{*j*} can only avoid some of the K_j ICIs it generated, for some of its effective transmit variables to avoid the ICI, setting $J(\bar{H}_0) = D^{\mathcal{W}}$ require $\bar{h}_{0, i_k, j}^{(2), t} = 1, \exists t$, while for other variables not to avoid the ICI, setting $J(\bar{H}_0) = D^{\mathcal{W}}$ require $\bar{h}_{0, i_k, j}^{(2), t} = 0, \forall t$. Now we see that the requirement

for the BS to avoid a part of the ICI is conflicted with the repeated feature of the Jacobian matrix. Consequently, the method in [11], [17] cannot be applied for this kind of MIMO-IBC. ■

Now we compare *Lemma 1* and *Lemma 2*, where the ways to eliminate the ICI from one BS to one user are different. For the class of MIMO-IBC considered in *Lemma 1*, the ICI can be eliminated either by only using the BS's (or the user's) spatial resources or by sharing the BS's and the user's spatial resources. That is to say, the ICI can be avoided by the BS or canceled by the user, which does not violate the repeated feature of $\mathbf{J}(\bar{\mathbf{H}}_0)$. Therefore, the feasibility of this class of MIMO-IBC can be extended from the that for MIMO-IC in [11]. For another class of MIMO-IBC in *Lemma 2*, part of the ICI can only be eliminated by sharing the resources, which is conflicted with the non-generic feature of $\mathbf{J}(\bar{\mathbf{H}}_0)$, hence we need to find other approach to construct an invertible Jacobian matrix.

2) *Proof of the sufficient condition in Theorem 2:* *Proof:* To show the structure of Jacobian matrix for the MIMO-IBC, we divide all the ICI into $\sum_{j=1}^G K_j$ groups, i.e., $\mathcal{Y} = \cup_{j=1}^G \cup_{l=1}^{K_j} \mathcal{Y}_{jl}$, where $\mathcal{Y}_{jl} = \{F_{i_k, jl} | k = 1, \dots, K_i, i = 1, \dots, G, i \neq j\}$ is a subset of the ICI generated from the effective transmit vector $\bar{\mathbf{v}}_{jl}$ of BS_{*j*}. Then as shown in Fig. 2, \mathbf{J} can be partitioned into $\sum_{i=1}^G K_i$ blocks, i.e., $\mathbf{J} = [\mathbf{J}_1^T, \dots, \mathbf{J}_G^T]^T$, $\mathbf{J}_j = [\mathbf{J}_{j1}^T, \dots, \mathbf{J}_{jK_j}^T]^T$, where the rows of the j_l th block $\mathbf{J}_{jl} \in \mathbb{C}^{\sum_{i=1, i \neq j}^G K_i \times L_v}$ correspond to the ICI generated by $\bar{\mathbf{v}}_{jl}$.

From (17) we know that \mathbf{J}_{jl} can be partitioned into $\mathbf{J}_{jl} = [\mathbf{J}_{jl}^V, \mathbf{J}_{jl}^U]$, where $\mathbf{J}_{jl}^V = \partial \text{vec}\{\mathcal{Y}_{jl}\} / \partial \text{vec}\{\bar{\mathbf{V}}\}$ and $\mathbf{J}_{jl}^U = \partial \text{vec}\{\mathcal{Y}_{jl}\} / \partial \text{vec}\{\bar{\mathbf{U}}\}$, $\text{vec}\{\bar{\mathbf{V}}\} = [\bar{\mathbf{v}}_{11}^T, \dots, \bar{\mathbf{v}}_{GK_G}^T]^T$ and $\text{vec}\{\bar{\mathbf{U}}\} = [\bar{\mathbf{u}}_{11}^H, \dots, \bar{\mathbf{u}}_{GK_G}^H]^T$ are comprised of all the effective transmit and receive vectors. Furthermore, \mathbf{J}_{jl}^V can be divided into $\sum_{i=1}^G K_i$ blocks, i.e., $\mathbf{J}_{jl}^V = [\mathbf{J}_{jl,11}^V, \dots, \mathbf{J}_{jl,GK_G}^V]$, where $\mathbf{J}_{jl,i_k}^V = \partial \text{vec}\{\mathcal{Y}_{jl}\} / \partial \bar{\mathbf{v}}_{i_k} \in \mathbb{C}^{\sum_{i=1, i \neq j}^G K_i \times (M_i - K_i)}$, whose rows correspond to all the ICI generated from $\bar{\mathbf{v}}_{jl}$ and whose columns correspond to all the variables provided by $\bar{\mathbf{v}}_{i_k}$.

Since \mathcal{Y}_{jl} is the ICI generated from $\bar{\mathbf{v}}_{jl}$, which can only be avoided by $\bar{\mathbf{v}}_{jl}$, from (24a) we obtain

$$\mathbf{J}_{jl,i_k}^V(\bar{\mathbf{H}}_0) = \begin{cases} \bar{\mathbf{H}}_{0:,j}^{(2)}, & \forall i_k = j_l \\ \mathbf{0}_{\sum_{i=1, i \neq j}^G K_i \times (M_i - K_i)}, & \text{otherwise.} \end{cases} \quad (33)$$

where $\bar{\mathbf{H}}_{0:,j}^{(2)} = [(\bar{\mathbf{h}}_{0,11,j}^{(2)})^T, \dots, (\bar{\mathbf{h}}_{0,(j-1)K_{(j-1)},j}^{(2)})^T, (\bar{\mathbf{h}}_{0,(j+1)1,j}^{(2)})^T, \dots, (\bar{\mathbf{h}}_{0,GK_G,j}^{(2)})^T]^T$.

It follows that $\mathbf{J}^V(\bar{\mathbf{H}}_0) = \text{diag}\{\mathbf{J}_{11,11}^V(\bar{\mathbf{H}}_0), \dots, \mathbf{J}_{GK_G,GK_G}^V(\bar{\mathbf{H}}_0)\}$, but $\mathbf{J}_{j1,j1}^V(\bar{\mathbf{H}}_0) = \dots = \mathbf{J}_{jK_j,jK_j}^V(\bar{\mathbf{H}}_0) = \bar{\mathbf{H}}_{0:,j}^{(2)}$, i.e., $\mathbf{J}_{j1,j1}^V(\bar{\mathbf{H}}_0)$ has K_j repeated blocks, which are marked with the same kind of shadowing field in Fig. 2. Now we see that in the Jacobian matrix the blocks corresponding

to the transmit vectors from each BS are identical, but the blocks corresponding to the receive vectors of the users are different. This comes from the second feature of MIMO-IBC.

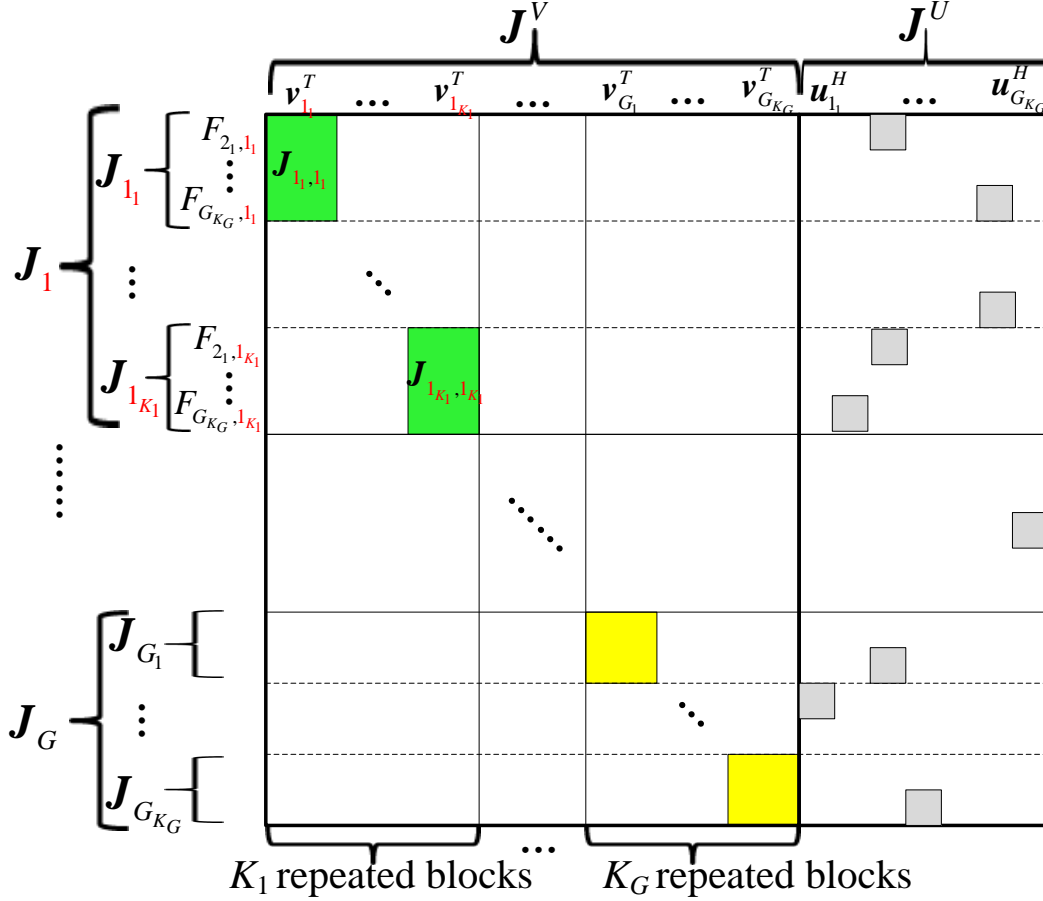


Fig. 2. Structure of $J(\bar{H}_0)$ of MIMO-IBC $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, 1))$.

Such a structure gives rise to the following observation: the transmit vector design for the BS in MIMO-IBC is not as flexible as the receive vector design for the user. If the transmit vector of BS_{*j*} for one of its own user needs to avoid the ICI to a user in other cell, the transmit vectors of the BS for its other users have also to avoid the ICI to the same user in other cell. This leads to the difficulty to construct a nonsingular Jacobian matrix for the MIMO-IBC shown in *Lemma 2*, where the BS can only avoid partial ICI it generated but it does not know which ICI it should avoid. Fortunately, the receive vector design for the user in a MIMO-IBC with $d = 1$ is flexible, because $J^U(\bar{H}_0)$ has the same structure as in MIMO-IC.

Inspired by this observation, we can first construct Jacobian matrix for the receive vector, i.e.,

assign its variables to deal with some ICI, using the way of perfect matching. Then, we construct Jacobian matrix for the transmit vectors to deal with the remaining ICI. To allow the transmit vectors of each BS for different local users to avoid different ICI, we only reserve enough variables in these transmit vectors but do not assign variables to eliminate specific ICI, considering the repeated nature of the Jacobian matrix. This translates to the following two rules to construct the Jacobian matrix.

- *Rule 1:* All elements in $\mathbf{J}^U(\bar{\mathbf{H}}_0)$ are set as the corresponding elements in \mathbf{D}^W directly.
- *Rule 2:* All elements in $\mathbf{J}_{j_1, j_1}^V(\bar{\mathbf{H}}_0)$ are set to ensure that its arbitrary $M_j - K_j$ row vectors are independent, and all elements in $\mathbf{J}_{j_l, j_l}^V(\bar{\mathbf{H}}_0)$ are set as $\mathbf{J}_{j_l, j_l}^V(\bar{\mathbf{H}}_0) = \mathbf{J}_{j_1, j_1}^V(\bar{\mathbf{H}}_0)$, $l = 2, \dots, K_j$.

Since $\mathbf{J}^V(\bar{\mathbf{H}}_0)$ is a block diagonal matrix, the nonzero blocks in different $\mathbf{J}_{j_l}^V(\bar{\mathbf{H}}_0)$ are non-overlapping. Since the entries in $\mathbf{J}^U(\bar{\mathbf{H}}_0)$ are set from \mathbf{D}^W , there is at most one nonzero element in each column or row of $\mathbf{J}^U(\bar{\mathbf{H}}_0)$. This indicates that the nonzero elements in different $\mathbf{J}_{j_l}^U(\bar{\mathbf{H}}_0)$ are also non-overlapping. As a result, the nonzero elements in different blocks of $\mathbf{J}_{j_l}(\bar{\mathbf{H}}_0) = [\mathbf{J}_{j_l}^V(\bar{\mathbf{H}}_0), \mathbf{J}_{j_l}^U(\bar{\mathbf{H}}_0)]$ are non-overlapping. Considering that $\mathbf{J}(\bar{\mathbf{H}}_0) = [\mathbf{J}_1^T(\bar{\mathbf{H}}_0), \dots, \mathbf{J}_G^T(\bar{\mathbf{H}}_0)]^T$, and $\mathbf{J}_j(\bar{\mathbf{H}}_0) = [\mathbf{J}_{j_1}^T(\bar{\mathbf{H}}_0), \dots, \mathbf{J}_{j_{K_j}}^T(\bar{\mathbf{H}}_0)]^T$, we have

$$\text{rank}(\mathbf{J}(\bar{\mathbf{H}}_0)) = \sum_{j=1}^G \sum_{l=1}^{K_j} \text{rank}(\mathbf{J}_{j_l}(\bar{\mathbf{H}}_0)) \quad (34)$$

In *Rule 1*, the perfect matching ensures that each interference can be eliminated by a unique variable of the effective receive vectors at the users. Since $\mathbf{J}_{j_l}(\bar{\mathbf{H}}_0)$ corresponds to all the ICI generated from the effective transmit vector $\bar{\mathbf{v}}_{j_l}$ of BS_{*j*}, whose total number is $\sum_{i=1, i \neq j}^G K_i$, there should be overall $\sum_{i=1, i \neq j}^G K_i$ variables in the transmit vector $\bar{\mathbf{v}}_{j_l}$ and receive vectors $\bar{\mathbf{u}}_{i_k}$ altogether to deal with these ICIs, $\forall i \neq j$. Since transmit vector $\bar{\mathbf{v}}_{j_l}$ can provide $M_j - K_j$ variables, receive vectors $\bar{\mathbf{u}}_{i_k}$ should provide $\sum_{i=1, i \neq j}^G K_i - (M_j - K_j) = \sum_{i=1}^G K_i - M_j$ variables, $\forall i \neq j$, to ensure the proper condition. This indicates that the perfect matching should ensure that there are $\sum_{i=1}^G K_i - M_j$ ones in $\mathbf{J}_{j_l}^U(\bar{\mathbf{H}}_0)$ which are scattered in different rows, and then $\text{rank}(\mathbf{J}_{j_l}^U(\bar{\mathbf{H}}_0)) = \sum_{i=1}^G K_i - M_j$. Using elementary transform, it is not difficult to eliminate $\sum_{i=1}^G K_i - M_j$ row vectors of $\mathbf{J}_{j_l}^V(\bar{\mathbf{H}}_0)$ and leave $M_j - K_j$ independent row vectors in $\mathbf{J}_{j_l}^V(\bar{\mathbf{H}}_0)$. In this way, the nonzero elements in $\mathbf{J}_{j_l}^V(\bar{\mathbf{H}}_0)$ and $\mathbf{J}_{j_l}^U(\bar{\mathbf{H}}_0)$ are located in different rows of $\mathbf{J}_{j_l}(\bar{\mathbf{H}}_0)$. Therefore, $\text{rank}(\mathbf{J}_{j_l}^V(\bar{\mathbf{H}}_0)) = M_j - K_j$ and $\text{rank}(\mathbf{J}_{j_l}(\bar{\mathbf{H}}_0)) = \text{rank}(\mathbf{J}_{j_l}^V(\bar{\mathbf{H}}_0)) + \text{rank}(\mathbf{J}_{j_l}^U(\bar{\mathbf{H}}_0)) = \sum_{i=1, i \neq j}^G K_i$. After substituting to (34), we

have

$$\text{rank}(\mathbf{J}(\bar{\mathbf{H}}_0)) = \sum_{j=1}^G \sum_{l=1}^{K_j} \sum_{i=1, i \neq j}^G K_i = \sum_{j=1}^G \sum_{i=1, i \neq j}^G K_j K_i \quad (35)$$

i.e., the constructed $\mathbf{J}(\bar{\mathbf{H}}_0)$ is nonsingular.

Now the theorem is proved. ■

3) *An example:* To understand the procedure of our proof, we consider a $(4 \times (2, 1)^2)(4 \times 2, 1)$ MIMO-IBC as an example, where there are 3 cells. In this MIMO-IBC system, the first and second cells support two users and the third cell supports one user, each user has one data stream, and each BS and user are equipped with four and two antennas, respectively. The effective transmit, receive and channel vectors are respectively

$$\begin{aligned} \bar{\mathbf{v}}_{i_k} &= \begin{cases} [\bar{v}_{i_k(1)} \ \bar{v}_{i_k(2)}]^T \in \mathbb{C}^{2 \times 1} & i_k = 1_1, 1_2, 2_1, 2_2 \\ [\bar{v}_{i_k(1)} \ \bar{v}_{i_k(2)} \ \bar{v}_{i_k(3)}]^T \in \mathbb{C}^{3 \times 1} & i_k = 3_1 \end{cases} \\ \bar{\mathbf{u}}_{i_k} &= [\bar{u}_{i_k(1)}] \in \mathbb{C}, \quad i_k = 1_1, 1_2, 2_1, 2_2, 3_1 \\ \bar{\mathbf{h}}_{0 \ i_k, j}^{(2)} &= \begin{cases} [\bar{h}_{i_k, j}^{(2),1} \ \bar{h}_{i_k, j}^{(2),2}] \in \mathbb{C}^{1 \times 2} & i_k = 1_1, 1_2, 2_1, 2_2 \\ [\bar{h}_{i_k, j}^{(2),1} \ \bar{h}_{i_k, j}^{(2),2} \ \bar{h}_{i_k, j}^{(2),3}] \in \mathbb{C}^{1 \times 3} & i_k = 3_1 \end{cases} \\ \bar{\mathbf{h}}_{0 \ i_k, j_t}^{(3)} &= [\bar{h}_{i_k, j_t}^{(3),1}] \in \mathbb{C}, \quad i_k = 1_1, 1_2, 2_1, 2_2, 3_1 \end{aligned}$$

The corresponding ICI-free transmission equations are

$$\begin{aligned} F_{2_1, 1_1}(\bar{\mathbf{H}}_0) &= \bar{u}_{2_1(1)}^* \bar{h}_{2_1, 1_1}^{(3),1} + \bar{h}_{2_1, 1}^{(2),1} \bar{v}_{1_1(1)} + \bar{h}_{2_1, 1}^{(2),2} \bar{v}_{1_1(2)} = 0 \\ F_{2_2, 1_1}(\bar{\mathbf{H}}_0) &= \bar{u}_{2_2(1)}^* \bar{h}_{2_2, 1_2}^{(3),1} + \bar{h}_{2_2, 1}^{(2),1} \bar{v}_{1_1(1)} + \bar{h}_{2_2, 1}^{(2),2} \bar{v}_{1_1(2)} = 0 \\ &\dots \\ F_{2_2, 3_1}(\bar{\mathbf{H}}_0) &= \bar{u}_{2_2(1)}^* \bar{h}_{2_2, 3_1}^{(3),1} + \bar{h}_{2_2, 3}^{(2),1} \bar{v}_{3_1(1)} + \bar{h}_{2_2, 3}^{(2),2} \bar{v}_{3_1(2)} + \bar{h}_{2_2, 3}^{(2),3} \bar{v}_{3_1(3)} = 0 \end{aligned} \quad (36)$$

Let $\mathcal{Y} = \{F_{2_1, 1_1}, F_{2_2, 1_1}, F_{3_1, 1_1}, F_{2_1, 1_2}, F_{2_2, 1_2}, F_{3_1, 1_2}, F_{1_1, 2_1}, F_{1_2, 2_1}, F_{3_1, 2_1}, F_{1_1, 2_2}, F_{1_2, 2_2}, F_{3_1, 2_2}, F_{1_1, 3_1}, F_{1_2, 3_1}, F_{2_1, 3_1}, F_{2_2, 3_1}\}$ and $\mathcal{X} = \mathcal{X}^V \cup \mathcal{X}^U$ denote the sets of all scalar equations and variables, where $\mathcal{X}^V = \{\bar{v}_{1_1(1)}, \bar{v}_{1_1(2)}, \bar{v}_{1_2(1)}, \bar{v}_{1_2(2)}, \bar{v}_{2_1(1)}, \bar{v}_{2_1(2)}, \bar{v}_{2_2(1)}, \bar{v}_{2_2(2)}, \bar{v}_{3_1(1)}, \bar{v}_{3_1(2)}, \bar{v}_{3_1(3)}\}$, and $\mathcal{X}^U = \{\bar{u}_{1_1(1)}, \bar{u}_{1_2(1)}, \bar{u}_{2_1(1)}, \bar{u}_{2_2(1)}, \bar{u}_{3_1(1)}\}$.

To illustrate the relationship between the elements of Jacobian matrix and the variables and equations, we show the Jacobian matrix in Table I, where the repeated block is in the same kind of shadowing.

TABLE I
JACOBIAN MATRIX OF $(4 \times (2, 1)^2)(4 \times 2, 1)$ MIMO-IBC

	$\bar{v}_{11}(1) \bar{v}_{11}(2)$	$\bar{v}_{12}(1) \bar{v}_{12}(2)$	$\bar{v}_{21}(1) \bar{v}_{21}(2)$	$\bar{v}_{22}(1) \bar{v}_{22}(2)$	$\bar{v}_{31}(1) \bar{v}_{31}(2) \bar{v}_{31}(3)$	$\bar{u}_{11}^*(1)$	$\bar{u}_{12}^*(1)$	$\bar{u}_{21}^*(1)$	$\bar{u}_{22}^*(1)$	$\bar{u}_{31}^*(1)$
$F_{21,11}$	$\bar{h}_{21,1}^{(2),1} \bar{h}_{21,1}^{(2),2}$							$\bar{h}_{21,11}^{(3),1}$		
$F_{22,11}$	$\bar{h}_{22,1}^{(2),1} \bar{h}_{22,1}^{(2),2}$								$\bar{h}_{22,11}^{(3),1}$	
$F_{31,11}$	$\bar{h}_{31,1}^{(2),1} \bar{h}_{31,1}^{(2),2}$									$\bar{h}_{31,11}^{(3),1}$
$F_{21,12}$		$\bar{h}_{21,1}^{(2),1} \bar{h}_{21,1}^{(2),2}$						$\bar{h}_{21,12}^{(3),1}$		
$F_{22,12}$		$\bar{h}_{22,1}^{(2),1} \bar{h}_{22,1}^{(2),2}$							$\bar{h}_{22,12}^{(3),1}$	
$F_{31,12}$		$\bar{h}_{31,1}^{(2),1} \bar{h}_{31,1}^{(2),2}$								$\bar{h}_{31,12}^{(3),1}$
$F_{11,21}$			$\bar{h}_{11,2}^{(2),1} \bar{h}_{11,2}^{(2),2}$			$\bar{h}_{11,21}^{(3),1}$				
$F_{12,21}$			$\bar{h}_{12,2}^{(2),1} \bar{h}_{12,2}^{(2),2}$				$\bar{h}_{12,21}^{(3),1}$			
$F_{31,21}$			$\bar{h}_{31,2}^{(2),1} \bar{h}_{31,2}^{(2),2}$							$\bar{h}_{31,21}^{(3),1}$
$F_{11,22}$				$\bar{h}_{11,2}^{(2),1} \bar{h}_{11,2}^{(2),2}$		$\bar{h}_{11,22}^{(3),1}$				
$F_{12,22}$				$\bar{h}_{12,2}^{(2),1} \bar{h}_{12,2}^{(2),2}$			$\bar{h}_{12,22}^{(3),1}$			
$F_{31,22}$				$\bar{h}_{31,2}^{(2),1} \bar{h}_{31,2}^{(2),2}$						$\bar{h}_{31,22}^{(3),1}$
$F_{11,31}$					$\bar{h}_{11,3}^{(2),1} \bar{h}_{11,3}^{(2),2} \bar{h}_{11,3}^{(2),3}$	$\bar{h}_{11,31}^{(3),1}$				
$F_{12,31}$					$\bar{h}_{12,3}^{(2),1} \bar{h}_{12,3}^{(2),2} \bar{h}_{12,3}^{(2),3}$		$\bar{h}_{12,31}^{(3),1}$			
$F_{21,31}$					$\bar{h}_{21,3}^{(2),1} \bar{h}_{21,3}^{(2),2} \bar{h}_{21,3}^{(2),3}$			$\bar{h}_{21,31}^{(3),1}$		
$F_{22,31}$					$\bar{h}_{22,3}^{(2),1} \bar{h}_{22,3}^{(2),2} \bar{h}_{22,3}^{(2),3}$				$\bar{h}_{22,31}^{(3),1}$	

A bipartite graph representing (36) is plotted by the dash lines in Fig. 3. It is not difficult to check that the considered MIMO-IBC is proper. Consequently, there exists at least one perfect matching, which can be obtained by the classic augmenting path algorithm in [18] and denoted by \mathcal{W} . This perfect matching is shown by the solid lines in Fig. 3 and its adjacency matrix is shown in (37),

denoted by $\mathbf{D}^{\mathcal{W}}$.

$$\mathbf{D}^{\mathcal{W}} = \begin{bmatrix} 1 & 0 & & & & & & & 0 & & \\ & 0 & 1 & & & & & & & 0 & \\ & \underline{0} & 0 & & & & & & & & 1 \\ & & & 0 & 0 & & & & & 1 & \\ & & & 0 & 1 & & & & & 0 & \\ & & & \underline{1} & 0 & & & & & & 0 \\ & & & & 1 & 0 & & & 0 & & \\ & & & & 0 & 0 & & & 1 & & \\ & & & & 0 & 1 & & & & & 0 \\ & & & & & 0 & 0 & & 1 & & \\ & & & & & 0 & 1 & & & 0 & \\ & & & & & 1 & 0 & & & & 0 \\ & & & & & & 1 & 0 & 0 & 0 & \\ & & & & & & 0 & 1 & 0 & & 0 \\ & & & & & & 0 & 0 & 1 & & 0 \\ & & & & & & 0 & 0 & 0 & & 1 \end{bmatrix} \quad (37)$$

Considering that $K_1 = K_2 = 2$ and $K_3 = 1$ in this MIMO-IBC, we have $N_{3_1} - 1 = 1$ and $\phi_3 = \{K_1, K_2, K_1 + K_2\} = \{2, 4\}$. Since $N_{3_1} - 1 > 0$ and $N_{3_1} - 1 \notin \phi_3$ which satisfy the conditions considered in *Lemma 2*, we know that neither the ICI from BS₁ to MS_{3₁} nor the ICI from BS₂ to MS_{3₁} can be canceled by MS_{3₁} independently. Therefore, there exists one BS whose ICI to MS_{3₁} must be eliminated by the BS and MS_{3₁}. Without loss of generality, we assume that the two ICI generated from BS₁ to MS_{3₁}, $F_{3_1,1_1}$ and $F_{3_1,1_2}$, need to be removed by BS₁ and MS_{3₁} jointly. According to the perfect matching shown in Fig. 3 and its adjacency matrix in (37), we assign $\bar{u}_{3_1(1)}^*$ and $\bar{v}_{1_2(1)}$ to cancel $F_{3_1,1_1}$ and avoid $F_{3_1,1_2}$, respectively. Since $\bar{v}_{1_2(1)}$ is assigned to avoid $F_{1_2,3_1}$, the (6,3)th element in $\mathbf{D}^{\mathcal{W}}$ is 1. Let the (6,3)th element in $\mathbf{J}(\bar{\mathbf{H}}_0)$ be 1, it requires $\bar{h}_{3_1,1}^{(2),1} = 1$. Moreover, since $\bar{u}_{3_1(1)}^*$ is assigned to cancel $F_{3_1,1_1}$, i.e., $\bar{v}_{1_1(1)}$ is not assigned to avoid $F_{1_1,3_1}$, the (3,1)th element in $\mathbf{D}^{\mathcal{W}}$ is 0. Let the (3,1)th element in $\mathbf{J}(\bar{\mathbf{H}}_0)$ be 1, it requires $\bar{h}_{3_1,1}^{(2),1} = 0$.⁷

⁷For the reader's convenience, we highlight the considered variables with underline in both (37) and Table I.

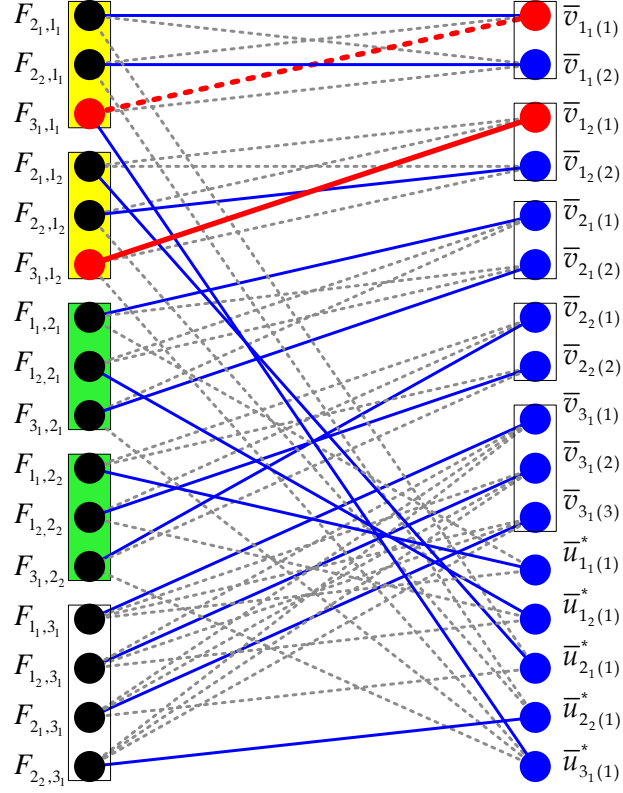


Fig. 3. The bipartite graph and a perfect matching.

Since it is impossible to set one variable as different values, we can not construct the nonsingular Jacobian matrix for this MIMO-IBC only by perfect matching as in [11].

In our proof, according to *Rule 1* and (37), we can set

$$\begin{aligned}
 \bar{h}_{11,22}^{(3)} &= \bar{h}_{12,21}^{(3)} = \bar{h}_{21,12}^{(3)} = \bar{h}_{22,31}^{(3)} = \bar{h}_{31,11}^{(3)} = 1 \\
 \bar{h}_{11,21}^{(3)} &= \bar{h}_{11,31}^{(3)} = \bar{h}_{12,22}^{(3)} = \bar{h}_{12,31}^{(3)} = \bar{h}_{21,11}^{(3)} \\
 &= \bar{h}_{21,31}^{(3)} = \bar{h}_{22,11}^{(3)} = \bar{h}_{22,12}^{(3)} = \bar{h}_{31,12}^{(3)} = \bar{h}_{31,21}^{(3)} = \bar{h}_{31,22}^{(3)} = 0
 \end{aligned}$$

From *Rule 2*, we set

$$\begin{bmatrix} \bar{h}_{21,1}^{(2),1} & \bar{h}_{21,1}^{(2),2} \\ \bar{h}_{22,1}^{(2),1} & \bar{h}_{22,1}^{(2),2} \\ \bar{h}_{31,1}^{(2),1} & \bar{h}_{31,1}^{(2),2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \bar{h}_{11,2}^{(2),1} & \bar{h}_{11,2}^{(2),2} \\ \bar{h}_{12,2}^{(2),1} & \bar{h}_{12,2}^{(2),2} \\ \bar{h}_{31,2}^{(2),1} & \bar{h}_{31,2}^{(2),2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \bar{h}_{11,3}^{(2),1} & \bar{h}_{11,3}^{(2),2} & \bar{h}_{11,3}^{(2),3} \\ \bar{h}_{12,3}^{(2),1} & \bar{h}_{12,3}^{(2),2} & \bar{h}_{12,3}^{(2),3} \\ \bar{h}_{21,3}^{(2),1} & \bar{h}_{21,3}^{(2),2} & \bar{h}_{21,3}^{(2),3} \\ \bar{h}_{22,3}^{(2),1} & \bar{h}_{22,3}^{(2),2} & \bar{h}_{22,3}^{(2),3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

After substituting these blocks to Table I, the Jacobian matrix in the example MIMO-IBC system is constructed as

$$\mathbf{J}(\bar{\mathbf{H}}_0) = \begin{bmatrix} 1 & 0 & & & & & & & & & 0 & & & & \\ & 0 & 1 & & & & & & & & & 0 & & & \\ & & 1 & 1 & & & & & & & & & 1 & & \\ & & & 1 & 0 & & & & & & & 1 & & & \\ & & & & 0 & 1 & & & & & & & 0 & & \\ & & & & 1 & 1 & & & & & & & & 0 & \\ & & & & & 1 & 0 & & & & & 0 & & & \\ & & & & & 0 & 2 & & & & & 1 & & & \\ & & & & & 3 & 4 & & & & & & 0 & & \\ & & & & & & 1 & 0 & & & & 1 & & & \\ & & & & & & & 0 & 2 & & & & 0 & & \\ & & & & & & & 3 & 4 & & & & & 0 & \\ & & & & & & & & 1 & 0 & 0 & 0 & & & \\ & & & & & & & & & 0 & 1 & 0 & & & \\ & & & & & & & & & & 0 & 0 & 1 & & \\ & & & & & & & & & & & 1 & 1 & 1 & \end{bmatrix} \quad (38)$$

Using the column vectors of $\mathbf{J}^U(\bar{\mathbf{H}}_0)$ to perform elementary transform on the column vectors of $\mathbf{J}^V(\bar{\mathbf{H}}_0)$, the nonzero elements in the 3rd, 4th, 8th, 10th and 15th row vectors of $\mathbf{J}^V(\bar{\mathbf{H}}_0)$ can be eliminated. After elementary row transform, (38) becomes a matrix who has the same nonzero

pattern as (37), which is a permutation matrix. Therefore, such a constructed $\mathbf{J}(\bar{\mathbf{H}}_0)$ is nonsingular, which means that this MIMO-IBC is feasible.

From (38) we can observe that the two blocks corresponding to the effective transmit matrices in each BS are identical. From (37) we can observe that:

- 1) the effective transmit vectors of the three BSs and the five effective receive vectors separately eliminate different ICIs in the rows,
- 2) all the ICIs are eliminated by the transmit and vectors. This helps to understand how the BSs and users “cooperate” implicitly.

V. DISCUSSION: PROPER VS FEASIBLE

In this section, we discuss the connection between proper condition and feasible condition of interference-free transmission for MIMO-IBC by analyzing the two theorems.

A. ‘Proper’=‘Feasible’

For a $\prod_{i=1}^G (M_i \times \prod_{k=1}^{K_i} (N_{i_k}, d))$ MIMO-IBC where $M_i \geq K_i d$ and $N_{i_k} \geq d$, from *Theorem 2* we know that when both M_i and N_{i_k} are divisible by d , the MIMO-IBC is feasible if it is proper. This immediately leads to the following conclusion: *when $d = 1$, a proper MIMO-IBC is feasible for arbitrary M_i and N_{i_k} .*

Since there are too many cases in asymmetric MIMO-IBC to describe and analyze, in the sequel we only focus on the cases in symmetric MIMO-IBC.

Corollary 1: For symmetric MIMO-IBC $(M \times (N, d)^K)^G$, the second necessary condition in *Theorem 1*, i.e., the proper condition in (3b), reduces to

$$M + N \geq (GK + 1)d \quad (39)$$

Before proving *Corollary 1*, we first briefly review the related results in the literature. By counting the total number of variables and equations, i.e., substituting $\mathcal{I} = \mathcal{J}$ and $\mathcal{K}_i = \{1, \dots, K\}$ into (3b), the following inequity is obtained in [14],

$$(M - Kd)GK + (N - d)GK \geq (G - 1)GK^2d \quad (40)$$

After simplifying (40), (39) is obtained. Therefore, (39) was proposed as one necessary condition for ICI-free transmission in MIMO-IBC [14]. From the definition of proper system in [10], a system is proper iff for *every* subset of equations, the number of the variables involved is at least as

large as the number of the equations. This means that to prove (39) as the proper condition, we need to check if (3b) always holds when (39) satisfies for *arbitrary* sets $\mathcal{I} \subseteq \mathcal{J}$ and $\mathcal{K}_i \subseteq \{1, \dots, K\}$.

Proof: For symmetric MIMO-IBC, (3b) becomes

$$(M - Kd) K_T + (N - d) K_R \geq \sum_{(i,j) \in \mathcal{I}} K |\mathcal{K}_i| d \quad (41)$$

where $K_T = \sum_{j \in \mathcal{I}_T} K$ and $K_R = \sum_{i \in \mathcal{I}_R} |\mathcal{K}_i|$, $\mathcal{I}_T = \{j | (i, j) \in \mathcal{I}\}$ and $\mathcal{I}_R = \{i | (i, j) \in \mathcal{I}\}$. \mathcal{I}_T and \mathcal{I}_R denote the index sets of the cells that generate ICI and suffer from the ICI in the cell pairs of \mathcal{I} , K_T and K_R are the total number of users in cells of \mathcal{I}_T and \mathcal{I}_R , respectively.

Define $\tilde{\mathcal{I}} \triangleq \{(i, j) | i \neq j, \forall j \in \mathcal{I}_T, i \in \mathcal{I}_R\}$, it is easy to know $\mathcal{I} \subseteq \tilde{\mathcal{I}}$.⁸ Therefore, the right-hand side of (41) satisfies

$$\begin{aligned} \sum_{(i,j) \in \mathcal{I}} K |\mathcal{K}_i| d &\leq \sum_{(i,j) \in \tilde{\mathcal{I}}} K |\mathcal{K}_i| d \\ &= \sum_{j \in \mathcal{I}_T} K \sum_{i \in \mathcal{I}_R, i \neq j} |\mathcal{K}_i| = \sum_{i \in \mathcal{I}_R} |\mathcal{K}_i| \sum_{j \in \mathcal{I}_T, j \neq i} K \end{aligned} \quad (42)$$

Since $\mathcal{I}_T \subseteq \{1, \dots, G\}$, we have $\sum_{i \in \mathcal{I}_R} |\mathcal{K}_i| \sum_{j \in \mathcal{I}_T, j \neq i} K \leq \sum_{i \in \mathcal{I}_R} |\mathcal{K}_i| \sum_{j=1, i \neq j}^G K = (G - 1) K K_R$. Since $\mathcal{I}_R \subseteq \{1, \dots, G\}$ and $|\mathcal{K}_i| \leq K$, we have

$$\sum_{j \in \mathcal{I}_T} K \sum_{i \in \mathcal{I}_R, i \neq j} |\mathcal{K}_i| \leq \sum_{j \in \mathcal{I}_T} K \sum_{i=1, i \neq j}^G K = (G - 1) K K_T$$

After substituting into (42), we obtain an upper-bound of the right-hand side of (41) as

$$\sum_{(i,j) \in \mathcal{I}} K |\mathcal{K}_i| d \leq (G - 1) K d \min\{K_R, K_T\} \quad (43)$$

Because $K_T \geq \min\{K_R, K_T\}$ and $K_R \geq \min\{K_R, K_T\}$, the left-hand side of (41) satisfies

$$(M - Kd) K_T + (N - d) K_R \geq (M + N - (K + 1)d) \min\{K_R, K_T\} \quad (44)$$

From (39), we have $M + N - (K + 1)d \geq (G - 1) K d$. Substituting this inequity into (44), we obtain a lower-bound of the left-hand side of (41) as

$$(M - Kd) K_T + (N - d) K_R \geq (G - 1) K d \min\{K_R, K_T\} \quad (45)$$

Consider (43) and (45), we obtain (41). ■

⁸For example, when $\mathcal{I} = \{(1, 3), (2, 4)\}$, we have $\mathcal{I}_R = \{1, 2\}$ and $\mathcal{I}_T = \{3, 4\}$. From the definition of $\tilde{\mathcal{I}}$, we know $\tilde{\mathcal{I}} = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$. Obviously, $\mathcal{I} \subseteq \tilde{\mathcal{I}}$.

From *Theorem 2* we know that for a $(M \times (N, d)^K)^G$ MIMO-IBC where $M \geq Kd$ and $N \geq d$, when M and N are divisible by d , the symmetric MIMO-IBC is feasible if it is proper. From *Corollary 1*, we can show that for a more general symmetric MIMO-IBC, it is feasible if it is proper.

Corollary 2: For a symmetric $(M \times (N, d)^K)^G$ MIMO-IBC, when

$$M \geq M_p, N \geq N_p, \exists p \in \{0, \dots, (G-1)K\} \quad (46)$$

where $M_p = (K+p)d$, $N_p = ((G-1)K + 1 - p)d$, $p = 0, \dots, (G-1)K$, the MIMO-IBC is feasible.

Proof: Since $M_p = (K+p)d$ and $N_p = ((G-1)K + 1 - p)d$, we have $M_p + N_p = (GK + 1)d$, $\forall p \in \{0, \dots, (G-1)K\}$. According to *Corollary 1*, we know that the MIMO-IBC $(M_p \times (N_p, d)^K)^G$ is proper.

Because $M_p \geq Kd$, $N_p \geq d$ and both M_p and N_p are divisible by d , the MIMO-IBC $(M_p \times (N_p, d)^K)^G$ is feasible according to *Theorem 2*.

For the MIMO-IBC $(M \times (N, d)^K)^G$ with arbitrary M and N that satisfy (46), we can always remove some redundant antennas and ensure that $M = M_p$ and $N = N_p$. Since the MIMO-IBC $(M_p \times (N_p, d)^K)^G$ is feasible, the MIMO-IBC $(M \times (N, d)^K)^G$ satisfying (46) must be feasible. ■

In *Corollary 2*, M and N may not be divisible by d or K . Since for the symmetric MIMO-IBC, the condition that either M or N is divisible by d is one special case of (46), it is easy to show that the proper MIMO-IBC $(M \times (N, d)^K)^G$ with either M or N is divisible by d is feasible.

To show the relationship of our result with existing results in literature regarding the proved sufficient conditions, we list the results in Table II. Although only MIMO-IC was considered in [11][12], their results can be extended into a special class of MIMO-IBC according to *Lemma 1*.

TABLE II
PROVED SUFFICIENT CONDITIONS FOR SYMMETRIC PROPER MIMO-IBC.

Considered or extended cases	Configurations
$\forall G, K$ in <i>Corollary 2</i>	$\begin{cases} M \geq (K+p)d \\ N \geq ((G-1)K + 1 - p)d \end{cases}$
$\forall G, K$ in [11]	M or N is divisible by Kd
$\forall G \geq 3, K$ in [12]	$M = N + (K-1)d$

The sufficiency proof in [11] can be extended to symmetric MIMO-IBC where either M or $N - d$ is divisible by Kd , which is a special case of that M or N is divisible by d , and is included in the cases shown in *Corollary 2*. The sufficiency proof in [12] can be extended to symmetric MIMO-IBC where $M = N + (K - 1)d$, which is equivalent to $M = N + (K - 1)d$, $M \geq Kd + (G - 1)Kd/2$ and $N \geq d + (G - 1)Kd/2$. When $(G - 1)K$ is even, let $p = (G - 1)K/2$, we have $M_p = Kd + (G - 1)Kd/2$ and $N_p = d + (G - 1)Kd/2$. It means that in this case the MIMO-IBC in *Corollary 2* is more general than the MIMO-IBC whose sufficiency proof can be extended from [12].

B. ‘Proper’ \neq ‘Feasible’

For $(M \times (N, d)^K)^G$ MIMO-IBC, the third necessary condition in *Theorem 1*, i.e., (3c), reduces to

$$\max\{pM, qN\} \geq pKd + qd \quad (47)$$

where $p \triangleq |\mathcal{I}_A|$ and $q \triangleq \sum_{i \in \mathcal{I}_B} |\mathcal{K}_i|$.

Since in (3c), $\mathcal{I}_A, \mathcal{I}_B \subseteq \{1, \dots, G\}$ and $\mathcal{I}_A \cap \mathcal{I}_B = \emptyset$, we have $\mathcal{I}_A \cup \mathcal{I}_B \subseteq \{1, \dots, G\}$ and $\mathcal{I}_A \cap \mathcal{I}_B = \emptyset$. Therefore, $|\mathcal{I}_A| \leq G - 1$, $|\mathcal{I}_B| \leq G - 1$ and $|\mathcal{I}_A| + |\mathcal{I}_B| \leq G$. From the definition of p and q , we can derive that,

$$\begin{cases} 1 \leq p \leq G - 1, 1 \leq q \leq (G - 1)K \\ Kp + q \leq GK \end{cases} \quad (48)$$

For a $(M \times (N, d)^K)^G$ MIMO-IBC where $M \geq Kd$ and $N \geq d$, when M and N satisfy (39) but do not satisfy (47), the MIMO-IBC is proper but infeasible.

Corollary 3: For a $(M \times (N, d)^K)^G$ MIMO-IBC where $M \geq Kd$ and $N \geq d$, there exist at least two proper but infeasible cases, which are

$$\text{Case I : } \begin{cases} \max\{M, (G - 1)KN\} < GKd \\ M + N \geq (GK + 1)d \end{cases} \quad (49a)$$

$$\text{Case II : } \begin{cases} \max\{(G - 1)M, N\} < ((G - 1)K + 1)d \\ M + N \geq (GK + 1)d \end{cases} \quad (49b)$$

When $G = 2$ and $K = 1$, Case I is the same as Case II, otherwise these two cases are different.

Proof: If M and N do not satisfy (47), we have $\max\{pM, qN\} < pKd + qd$, i.e., $pM < pKd + qd$ and $qN < pKd + qd$. Considering $M + N \geq (GK + 1)d$ in (39), we can obtain the proper but infeasible region, which satisfies

$$M < \frac{pK + q}{p}d, N < \frac{pK + q}{q}d \quad (50a)$$

$$M + N \geq (GK + 1)d \quad (50b)$$

From (50a), we have $M + N < (pK + q)(1/p + 1/q)d$. From (50b), we have $M + N \geq (GK + 1)d$. Therefore, only if $(pK + q)(1/p + 1/q) > GK + 1$, the proper but infeasible region is not empty. It is not hard to shown that in the nonempty region p, q need to satisfy the following quadratic inequality,

$$\Delta \triangleq K \left(\frac{p}{q} \right)^2 - (G - 1)K \frac{p}{q} + 1 > 0 \quad (51)$$

In (51), Δ is a convex function. Therefore, if (51) does not hold when the value of p/q achieves its minimum or maximum, it will not hold for other values of p and q .

To find the cases that are proper but infeasible, we first check whether (51) is satisfied when p/q achieves its minimum or maximum. From (48), it is easy to show that when $p = 1$, $q = (G - 1)K$, $p/q = 1/((G - 1)K)$ achieves the minimum, while when $p = (G - 1)$, $q = 1$, $p/q = (G - 1)$ is the maximum.

When $p = 1$, $q = (G - 1)K$, we have $\Delta = 1/((G - 1)^2K)$. Hence, (51) holds for all G, K . Substituting the values of p, q into (50a), we have $\max\{M, (G - 1)KN\} < GKd$. Combining with (50b), we obtain (49a), i.e., Case I.

When $p = (G - 1)$, $q = 1$, we have $\Delta = 1 > 0$. Consequently, (51) still holds for all G, K . Substituting the values of p, q into (50a), we have $\max\{(G - 1)M, N\} < ((G - 1)K + 1)d$. Combining with (50b), we obtain (49b), i.e., Case II. ■

Corollary 3 implies that for proper symmetric MIMO-IBC where $M \geq Kd$ and $N \geq d$, there exist other necessary conditions to ensure interference-free transmission. From the *Corollary*, we can immediately obtain two necessary conditions, which are $\max\{M, (G - 1)KN\} \geq GKd$ and $\max\{(G - 1)M, N\} \geq ((G - 1)K + 1)d$. The studies in [8], [11], [15], [19], [20] proposed necessary conditions other than the proper condition. To show the relationship of our result with theirs for symmetric MIMO-IBC, we list these results in Table III.

TABLE III
NECESSARY CONDITIONS FOR SYMMETRIC PROPER MIMO-IBC.

Considered or extended cases	Necessary conditions can be proved	Proper but infeasible cases we derived
$\forall G, K$ in <i>Corollary 3</i>	$\begin{cases} \max\{M, (G-1)KN\} \geq GKd \\ \max\{(G-1)M, N\} \geq ((G-1)K+1)d \end{cases}$	Case I and II, $\forall G, K$
$\forall G, K$ in [11] ⁸	$\max\{M, N\} \geq (K+1)d$	Case II, $\forall G = 2, K$
$\forall G, K$ in [15]	$\begin{cases} \max\{M, (L-1)KN\} \geq LKd \\ \max\{(L-1)M, KN\} \geq LKd \end{cases}, L = 2, \dots, G$	Case I, $\forall G, K$; Case II, $\forall G, K = 1$
$\forall G, K$ in [8]	$\begin{cases} \max\{M, (G-1)N\} \geq (K+G-1)d \\ \max\{(G-1)M, KN\} \geq (K+G-1)d \end{cases}$	Cases I and II, $\forall G, K = 1$
$G = 3, K = 1$ in [19], [20]	$\begin{cases} \max\{LM, (L+1)N\} \geq (2L+1)d \\ \max\{(L+1)M, LN\} \geq (K+G-1)d \end{cases}, \forall L \geq 1$	Cases I, II and others, $G = 3, K = 1$

We also list the corresponding proper but infeasible cases, which can be obtained from the necessary conditions after some regular but tedious derivations. For concise, we omit the details of the derivation. For general symmetric MIMO-IBC, most of the necessary conditions in [8], [11], [15] can be derived from the proper condition. For some special symmetric MIMO-IBC, e.g., $G = 2$ or $K = 1$, one or two necessary conditions can not be derived from the proper condition, which leads to one or two proper but infeasible cases. From Table III, we can see that these proper but infeasible cases except those in [19], [20] are all special cases of *Corollary 3*.

For the symmetric three-cell MIMO-IC, all the necessary conditions in [19], [20] can not be derived from the proper condition. When $L = 1$, the obtained proper but infeasible cases happen to be Cases I and II of our results. When $L > 1$, the necessary conditions lead to other proper but infeasible cases. Consequently, when $G = 3$ and $K = 1$, the necessary condition in [19], [20] is more general than ours.

C. An Example

In Fig. 4, we illustrate the feasible and infeasible regions in antenna configuration for an example MIMO-IBC. The feasible results from *Corollary 2* are shown by horizontal lines. The extended results from [11] through *Lemma 1* are in dash lines, and the extended results from [12] are in

⁸For the symmetric MIMO-IC, the study in [11] provided a necessary condition of $\max\{M, N\} \geq 2d$. By extending this condition into MIMO-IBC directly, it becomes $\max\{M, N\} \geq (K+1)d$.

dot-dash lines. We can see that the extended results from [11] are included in our results, and those from [12] are special cases of ours when $(G - 1)K$ is even.

The two proper but infeasible cases in *Corollary 3* are highlighted with darker shadowing. In the proper region, except for the region that has been proved to be feasible in (46) and that has been proved to be infeasible in (49a) and (49b), the feasibility of the remaining region is still unknown and will be investigated in our future work.

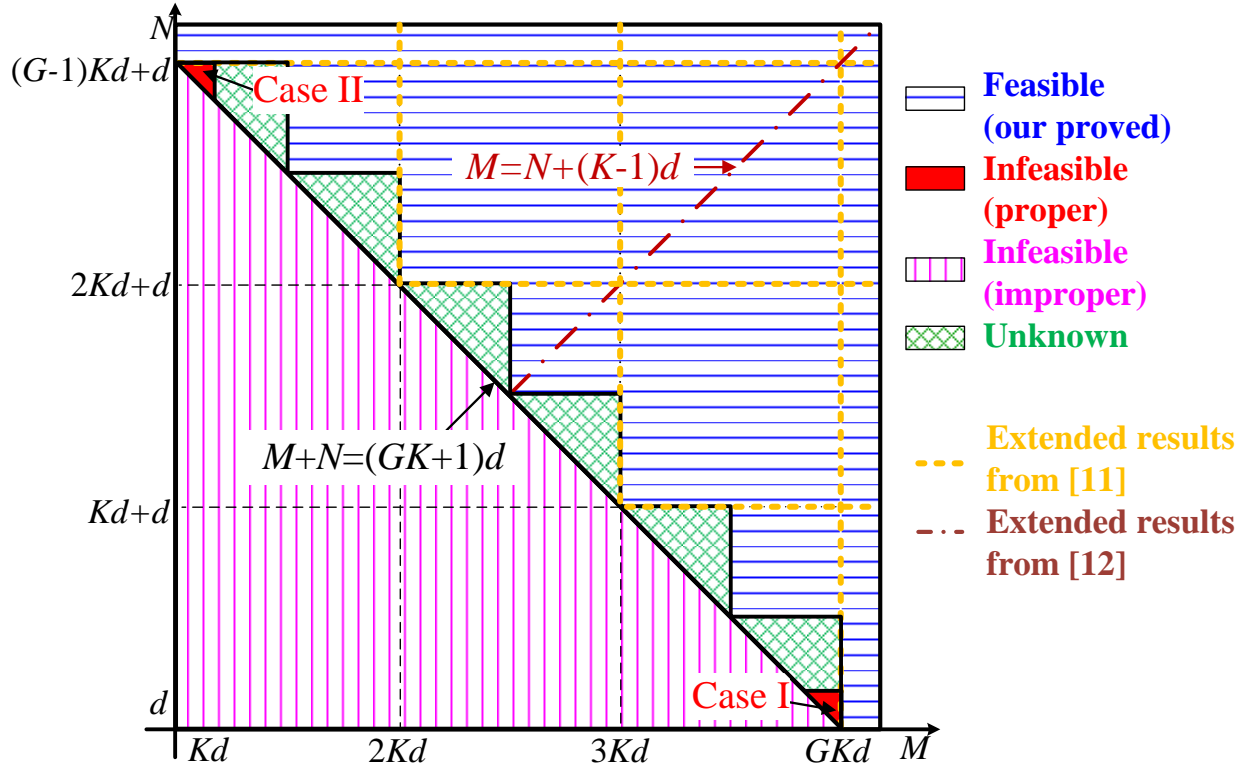


Fig. 4. Feasibility of MIMO-IBC to support overall GKd data streams, $G = 4$, $K = 2$, where $(G - 1)K$ is even.

VI. CONCLUSION

In this paper, we proposed and proved necessary conditions to support interference-free transmission for general MIMO-IBC without symbol extension. Except for proper condition, another condition was posed to ensure the elimination of a kind of irreducible interference. The existence conditions of the reducible and irreducible interference were provided, which depend on the difference in spatial dimension between a base station and multiple users or between a user and multiple base stations. The distinctive mechanisms of removing these two kind of interference were

discussed, which leads to the difference in the feasibility conditions for MIMO-IBC with various configurations and in the ways to prove the conditions. We found the classes of MIMO-IBC whose proof of sufficiency in interference alignment feasibility cannot be extended from that for MIMO-IC and whose can. To overcome the difficulty in proving the sufficient conditions of a system with non-generic coefficient matrices in the polynomial equations, we exploiting the flexibility of MIMO-IBC in designing the receive matrices at the users, and we proved the sufficiency for a special class of MIMO-IBC. Finally, we showed when a proper symmetric MIMO-IBC is feasible and when cannot.

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